Fatigue Damage Modelling of Fibre-reinforced Composite Materials: Review

Joris Degrieck and Wim Van Paepegem[*]

Department of Mechanical Construction and Production, Ghent University, Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium

This paper presents a review of the major fatigue models and life time prediction methodologies for fibre-reinforced polymer composites, subjected to fatigue loadings.

In this review, the fatigue models have been classified in three major categories: fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams and introduce some sort of fatigue failure criterion; phenomenological models for residual stiffness/strength; and finally progressive damage models which use one or more damage variables related to measurable manifestations of damage (transverse matrix cracks, delamination size).

Although this review does not pretend to be exhaustive, the most important models proposed during the last decades have been included, as well as the relevant equations upon which the respective models are based.

1 INTRODUCTION

As a result of their high specific stiffness and strength, fibre-reinforced composites are often selected for weight-critical structural applications. However deficiencies in current life time prediction methodologies for these materials often require large factors of safety to be adopted. Therefore composite structures are often overdesigned and extensive prototype-testing is required to allow for an acceptable life time prediction. Improved damage accumulation models and life time prediction methodologies may result in a more efficient use of these materials and in a shorter time-to-market.

2 FATIGUE DAMAGE MODELLING: GENERAL CONSIDERATIONS

In general fatigue of fibre-reinforced composite materials is a quite complex phenomenon, and a large research effort is being spent on it today.

Fibre-reinforced composites have a rather good rating as regards to life time in fatigue. The same does not apply to the number of cycles to initial damage nor to the evolution of damage. Composite materials are inhomogeneous and anisotropic, and their behaviour is more complicated than that of homogeneous and isotropic materials such as metals. The main reasons for this are the different types of damage that can occur (e.g. fibre fracture, matrix cracking, matrix crazing, fibre buckling, fibre-matrix interface failure, delaminations,...), their interactions and their different growth rates.
Among the parameters that influence the fatigue performance of composites are:
- fibre type,
- matrix type,
- type of reinforcement structure (unidirectional, mat, fabric, braiding,...),
- laminate stacking sequence,
- environmental conditions (mainly temperature and moisture absorption),
- loading conditions (stress ratio R, cycling frequency,...) and boundary conditions.

As a consequence the microstructural mechanisms of damage accumulation, of which there are several, occur sometimes independently and sometimes interactively, and the predominance of one or other of them may be strongly affected by both material variables and testing conditions.

There are a number of differences between the fatigue behaviour of metals and fibre-reinforced composites. In metals the stage of gradual and invisible deterioration spans nearly the complete life time. No significant

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[*] Author to whom correspondence should be addressed.

reduction of stiffness is observed during the fatigue process. The final stage of the process starts with the formation of small cracks, which are the only form of macroscopically observable damage. Gradual growth and coalescence of these cracks quickly produce a large crack and final failure of the structural component. As the stiffness of a metal remains quasi unaffected, the linear relation between stress and strain remains valid, and the fatigue process can be simulated in most common cases by a linear elastic analysis and linear fracture mechanics.

In a fibre-reinforced composite damage starts very early and the extent of the damage zones grows steadily, while the damage type in these zones can change (e.g. small matrix cracks leading to large size delaminations). The gradual deterioration of a fibre-reinforced composite – with a loss of stiffness in the damaged zones – leads to a continuous redistribution of stress and a reduction of stress concentrations inside a structural component. As a consequence an appraisal of the actual state or a prediction of the final state (when and where final failure is to be expected) requires the simulation of the complete path of successive damage states.

According to Fong (1982), there are two technical reasons why fatigue damage modelling in general is so difficult and expensive. The first reason are the several scales where damage mechanisms are present: from atomic level, through the subgrain, grain and specimen levels, to the component and structural levels. The second reason is the impossibility of producing 'identical' specimens with well-characterized microstructural features.

Fong also draws the attention to some pitfalls of fatigue damage modelling:
- confusion over scale: information from measurements on different scale levels, is combined improperly and leads to erroneous results,
- false generalization: for example stiffness reduction can often be divided in three regimes: sharp initial reduction – more gradual decrease – final failure (Schulte *et al* (1985), Daniel and Charewicz (1986)), but the related models are not always valid in the three stages,
- oversimplification: curve fitting of experimental data is done by using oversimplified expressions. This last statement was confirmed by Barnard *et al* (1985). He presented evidence that much of the scatter of the S-N curve drawn from his experimental data was caused by a change in failure mode, generating a discontinuity in the S-N curve. Indeed a Students *t*-distribution indicated that his test data were falling apart in two distinct and statistically significant populations. The remaining scatter was a consequence of static strength variations.

Next, many models have been established for laminates with a particular stacking sequence and particular boundary conditions, under uniaxial cyclic loading with constant amplitude, at one particular frequency,... The extrapolation to real structures with a stacking sequence varying from point to point, and more complex variations of the loads, is very complicated, if not impossible. Indeed some serious difficulties have to be overcome when fatigue life prediction of composite materials under general loading conditions is pursued:
- the governing damage mechanism is not the same for all stress level states (Barnard *et al* (1985), Daniel and Charewicz (1986)). Failure patterns vary with cyclic stress level and even with number of cycles to failure,
- the load history is important. When block loading sequences are applied in low-high order or in high-low order, there can be a considerable difference in damage growth (Hwang and Han (1986a)),
- most experiments are performed in uniaxial stress conditions (e.g. uniaxial tension/compression), although these stress states are rather exceptional in real structures,
- the residual strength and fatigue life of composite laminates have been observed to decrease more rapidly when the loading sequence is repeatedly changed after only a few loading cycles (Farrow (1989)). This so-called 'cycle-mix effect' shows that laminates that experience small cycle blocks, have reduced average fatigue lives as compared to laminates that are subjected to large cycle blocks, although the total number of cycles they have been subjected to, is the same for both laminates at the end of the experiment,
- the frequency can have a major impact on the fatigue life. Ellyin and Kujawski (1992) investigated the frequency effect on the tensile fatigue performance of glass fibre-reinforced $[\pm 45°]_{5S}$ laminates and concluded that there was a considerable influence of test loading frequency. Especially for matrix dominated laminates and loading conditions, frequency becomes important because of the general sensitivity of the matrix to the loading rate and because of the internal heat generation and associated temperature rise.

Clearly a lot of research has still to be done in this domain. However several attempts have been made to extend models for uniaxial constant amplitude loading to more general loading conditions, such as block-type and spectrum loading and to take into account the effect of cycling frequency and multiaxial loads.

3   REVIEW OF EXISTING FATIGUE MODELS

This review aims to outline the most important fatigue models and life time prediction methodologies for fatigue testing of fibre-reinforced polymers. A rigorous classification is difficult, but a workable classification can be based on the classification of fatigue criteria by Sendeckyj (1990). According to Sendeckyj, fatigue criteria can be classified in four major categories: the macroscopic strength fatigue criteria, criteria based on residual strength and those based on residual stiffness, and finally the criteria based on the actual damage mechanisms.
A similar classification has been used by the authors to classify the large number of existing fatigue models for composite laminates and consists of three major categories: fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams and introduce some sort of fatigue failure criterion; phenomenological models for residual stiffness/strength; and finally progressive damage models which use one or more damage variables related to measurable manifestations of damage (transverse matrix cracks, delamination size). The next paragraph briefly justifies the classification.

Although the fatigue behaviour of fibre-reinforced composites is fundamentally different from the behaviour exposed by metals, many models have been established which are based on the well-known S-N curves. These models make up the first class of so-called 'fatigue life models'. This approach requires extensive experimental work and does not take into account the actual damage mechanisms, such as matrix cracks and fibre fracture.
The second class comprises the phenomenological models for residual stiffness and strength. These models propose an evolution law which describes the (gradual) deterioration of the stiffness or strength of the composite specimen in terms of macroscopically observable properties, as opposed to the third class of progressive damage models, where the evolution law is proposed in direct relation with specific damage. Residual stiffness models account for the degradation of the elastic properties during fatigue. Stiffness can be measured frequently during fatigue experiments, and can be measured without further degrading the material (Highsmith and Reifsnider (1982)). The model may be deterministic, in which a single-valued stiffness property is predicted, or statistical, in which predictions are for stiffness distributions. The other approach is based on the composite's strength. In many applications of composite materials it is important to know the residual strength of the composite structure, and as a consequence the remaining life time during which the structure can bear the external load. Therefore the so-called 'residual strength' models have been developed, which describe the deterioration of the initial strength during fatigue life. From their early use, strength-based models have generally been statistical in nature. Most commonly, two-parameter Weibull functions are used to describe the residual strength and probability of failure for a set of laminates after an arbitrary number of cycles.
Since the damage mechanisms which govern the fatigue behaviour of fibre-reinforced composites, have been studied intensively during the last decades, a last class of models have been proposed which describe the deterioration of the composite material in direct relation with specific damage (e.g. transverse matrix cracks, delamination size). These models correlate one or more properly chosen damage variables to some measure of the damage extent, quantitatively accounting for the progression of the actual damage mechanisms. These models are often designated as 'mechanistic' models.
Summarized, fatigue models can be generally classified in three categories: the fatigue life models; the phenomenological models for residual stiffness/strength; and the progressive damage models.

One of the important outcomes of all established fatigue models is the life time prediction. Each of the three categories uses its own criterion for determining final failure and as a consequence for the fatigue life of the composite component.
The fatigue life models use the information from S-N curves or Goodman-type diagrams and introduce a fatigue failure criterion which determines the fatigue life of the composite specimen. Regarding the characterization of the S-N behaviour of composite materials, Sendeckyj (1981) advises to take into account three assumptions:
- the S-N behaviour can be described by a deterministic equation,
- the static strengths are uniquely related to the fatigue lives and residual strengths at runout (termination of cyclic testing). An example of such a relationship is the commonly used 'strength-life equal rank assumption' (Hahn and Kim (1975), Chou and Croman (1978)) which states that for a given specimen its rank in static strength is equal to its rank in fatigue life,
- the static strength data can be described by a two-parameter Weibull distribution.
Residual strength models have in fact an inherent 'natural failure criterion': failure occurs when the applied stress equals the residual strength (Harris (1985), Schaff and Davidson (1997a)). In the residual stiffness approach, fatigue failure is assumed to occur when the modulus has degraded to a critical level which has been defined by many investigators. Hahn and Kim (1976) and O'Brien and Reifsnider (1981) state that fatigue failure occurs when the fatigue secant modulus degrades to the secant modulus at the moment of failure in a static test.

According to Hwang and Han (1986a), fatigue failure occurs when the fatigue resultant strain reaches the static ultimate strain.

Damage accumulation models and life time prediction methodologies are very often inherently related, since the fatigue life can be predicted by establishing a fatigue failure criterion which is imposed to the damage accumulation model. For specific damage types, the failure value of the damage variable(s) can be determined experimentally.

For each fatigue model mentioned hereafter, it will be clearly stated if fatigue experiments have been conducted on notched specimens. Indeed some damage models are not applicable to notched specimens, because central holes and sharp notches at the edge of a specimen are known to be stress-concentrators. On the other hand such specimens are often used to deliberately initiate delaminations at a well-known site in the specimen.

Although excellent review papers on the fatigue behaviour of fibre-reinforced composites have been published in the past (Goetchius (1987), Reifsnider (1990), Stinchcomb and Bakis (1990), Sendeckyj (1990), Saunders and Clark (1993)), this paper intends to focus on the existing *modelling* approaches for the fatigue behaviour of fibre reinforced polymers. Since the vast majority of the fatigue models has been developed for and applied to a specific composite material and specific stacking sequence, it is very difficult to assess to which extent a particular model can be applied to another material type than the one it was tested for (glass/carbon fibre, thermoplastic/thermosetting matrix, unidirectional/woven/ stitched/braided reinforcement, unnotched/notched laminates,…), but this paper wants to give at least a comprehensive survey of the most important modelling strategies for fatigue behaviour. For an in-depth discussion of the fatigue models, illustrated with figures and experimental results, the reader is referred to the original publications cited in the references. The authors have chosen to preserve the style of the equations (symbols, notations,...) as it was used by the researchers themselves, because the familiarity of the reader with the commonly known models could be lost when changing the symbols and notations of the equations.

3.1 Fatigue life models

The first category contains the so-called 'fatigue life' models: these models extract information from the S-N curves or Goodman-type diagrams and propose a fatigue failure criterion. They do not take into account damage accumulation, but predict the number of cycles, at which fatigue failure occurs under fixed loading conditions.

One of the first fatigue failure criteria was proposed by Hashin and Rotem (1973). They distinguished a fibre-failure and a matrix-failure mode:

$$\sigma_A = \sigma_A^u$$
$$\left(\frac{\sigma_T}{\sigma_T^u}\right)^2 + \left(\frac{\tau}{\tau^u}\right)^2 = 1 \tag{1}$$

where σ_A and σ_T are the stresses along the fibres and transverse to the fibres, τ is the shear stress and σ_A^u, σ_T^u and τ^u are the ultimate tensile, transverse tensile and shear stress, respectively. Since the ultimate strengths are function of fatigue stress level, stress ratio and number of cycles, the criterion is expressed in terms of three S-N curves which must be determined experimentally from testing off-axis unidirectional specimens under uniaxial load.

This criterion is in fact only valid for laminates with unidirectional plies, under the further restriction that discrimination between the two failure modes exhibited during fatigue failure, should be possible.

Ellyin and El-Kadi (1990) demonstrated that the strain energy density can be used in a fatigue failure criterion for fibre-reinforced materials. The fatigue life N_f was related to the total energy input ΔW^t through a power law type relation of the form:

$$\Delta W^t = \kappa N_f^\alpha \tag{2}$$

where κ and α were shown to be functions of the fibre orientation angle. In comparison with experimental data from tests on glass/epoxy specimens, an expression for α and κ as a function of the fibre orientation angle was established.

The strain energy density was calculated under an elastic plane stress hypothesis. To include interlaminar shear and through-the-thickness stress distribution, another expression for the strain energy density should be derived.

Reifsnider and Gao (1991) established a fatigue failure criterion, based upon an average stress formulation of composite materials derived from the Mori-Tanaka method (a method to calculate the average stress fields in inhomogeneities and their surrounding matrix). The criterion is at the micromechanics level and takes into account the properties of the constituents and the interfacial bond.

Although very similar to the fatigue failure criteria proposed by Hashin and Rotem (1973), the failure criteria for matrix-dominated and fibre-dominated failure are expressed in terms of the average stresses $\langle \sigma_{ij}^m \rangle$ and $\langle \sigma_{ij}^f \rangle$ in the matrix and fibres, respectively. These average stresses are calculated by applying the Mori-Tanaka method, while taking into account the problem of non-perfectly bonded interfaces between fibres and matrix by modelling the interface as a thin layer with spring-like behaviour.

Finally, the failure functions for the two failure mechanisms are:

$$\left\langle \sigma_{11}^f \right\rangle = X^f$$

$$\left(\frac{\left\langle \sigma_{22}^m \right\rangle}{X^m} \right)^2 + \left(\frac{\left\langle \sigma_{12}^m \right\rangle}{S^m} \right)^2 = 1 \tag{3}$$

where X^f and X^m are fatigue failure functions under tensile loading for fibre and unreinforced matrix materials respectively, while S^m is the fatigue failure function of the unreinforced matrix under shear loading. These failure functions depend on the stress ratio R, the number of cycles N and the frequency f, and are actually S-N curves which should be determined experimentally in advance.

The micromechanics model was applied to off-axis fatigue loading of unidirectional E-glass/epoxy laminae. When simulating the experiments, the interface was assumed to be perfectly bonded to simplify the mathematics, although the theory was derived for non-perfectly bonded interfaces.

Lawrence Wu (1993) used a macroscopic failure criterion, based on the Tsai-Hill failure criterion. The criterion was expressed as:

$$\frac{3}{2(F+G+H)} \left[F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 \right.$$
$$\left. + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 + 2N\sigma_{xy}^2 \right] = \overline{\sigma}^2 \tag{4}$$

where F, G, H, L, M and N are functions of the lamina peak stresses X, Y, Z in the x, y and z directions and Q, R and S, which are the lamina shear stresses with respect to the shear stress components σ_{yz}, σ_{zx} and σ_{xy}, respectively. $\overline{\sigma}^2$ is an equivalent stress in terms of X, Y and Z. The peak stresses X, Y, Z, Q, R and S are all functions of fatigue life N_f and the corresponding S-N curves must be determined in advance.

Comparison was made with S-N data for $[\pm 45°]_s$ and $[0°/90°]_s$ carbon fibre-reinforced laminates from other investigators. The stresses were obtained from finite element analysis, taking into account free edge effects and initial thermal stresses due to curing. Inclusion of initial thermal stresses into the analysis appeared to improve the results.

Fawaz and Ellyin (1994) proposed a semi-log linear relationship between applied cyclic stress S and the number of cycles to failure N:

$$S = m \log(N) + b$$
$$S_r = m_r \log(N) + b_r \tag{5}$$

where the second equation applies to a well chosen reference line. The relation between the two sets of material parameters (m, b) and (m_r, b_r) is specified by:

$$m = f(a_1, a_2, \theta).g(R).m_r$$
$$b = f(a_1, a_2, \theta).b_r$$

(6)

where a_1 is the first biaxial ratio ($a_1 = \dfrac{\sigma_y}{\sigma_x}$), a_2 is the second biaxial ratio ($a_2 = \dfrac{\tau_{xy}}{\sigma_x}$), R is the stress ratio and θ is the stacking angle.

Their model could be generalized in the expression:

$$S(a_1, a_2, \theta, R, N) = f(a_1, a_2, \theta) \cdot \left[g(R) m_r \log(N) + b_r \right]$$

(7)

The aim of the model was then to predict the parameters m and b (related with m_r and b_r through the functions f and g) of a general S-log(N) line, for any a, θ and R.

Their model has been applied to a number of experimental studies that exist in the literature and the correlation is shown to be quite accurate. However the model seems to be rather sensitive to the choice of the reference line S_r.

Harris and his co-workers (Harris (1985), Adam *et al* (1994), Gathercole *et al* (1994)) who have performed extensive research on fatigue in composite materials, proposed a so-called normalized constant-life model that expresses which combinations of mean and peak stress amplitudes give rise to the same number of cycles to failure:

$$a = f.(1 - m)^u.(c + m)^v$$

(8)

where f, u and v are linear functions of 'log N_f' (f often kept constant); σ_t is the tensile strength, $a = \dfrac{\sigma_{alt}}{\sigma_t}$ is the normalized alternating stress component; $m = \dfrac{\sigma_m}{\sigma_t}$ is the normalized mean stress component and $c = \dfrac{\sigma_c}{\sigma_t}$ is the normalized compression strength. The exponents u and v are responsible for the shapes of the left and right wings of the bell-shaped curve, and as the two exponents do not need to be exactly equal, the curve can be asymmetric. The procedure for performing the constant-life analysis is described in detail by Harris (1985). The final output results in a family of predicted constant life curves.

In recent articles, Beheshty and Harris (1998) and Beheshty *et al* (1999) proved that their model can be applied to impact-damaged laminates as well. In that case, the left-hand (predominantly compression) quadrant of the constant-life diagram is substantially modified by the impact damage, through its effect in reducing the compression strength of the material, but the curve in the right-hand quadrant is much less affected.

It is worth to note that from these experiments, Beheshty *et al* (1999) concluded that the observed values of the parameter f appeared to depend on the normalized compression strength c, as opposed to former statements. They suggested an inverse power-law relationship between f and c with two constants being functions of fatigue life N_f.

Andersons and Korsgaard (1997) observed that creep accelerated under cyclic loading in the case of glass/polyester composites for use as a blade material for wind turbines. They used the life fraction as a measure of fatigue damage D, and the effect of fatigue damage on the viscoelastic response of the composite was modelled by a damage-dependent effective stress $\sigma_{ef} = \sigma \cdot (1 + c \cdot D)$, where σ is the applied stress, D = n/N is the life fraction and c is a constant, depending on stress ratio and applied stress level. The linear viscoelasticity relations were then:

$$\varepsilon(t) = \frac{\sigma_{ef}(t)}{E} + \int_0^t K(t-\tau)\sigma_{ef}(\tau)d\tau$$

$$K(t-\tau) = \sum_i \frac{a_i}{b_i} \exp\left(-\frac{t-\tau}{b_i}\right)$$

(9)

where a_i and b_i are creep parameters, determined from creep tests (stress ratio = 1) at low stress level.

It is important to note that test data showed that the fatigue strength tends to converge to the creep rupture strength when the mean stress is increased, instead of to the ultimate tensile strength as is routinely assumed when constructing the Goodman diagram.

Jen and Lee (1998a, 1998b) modified the Tsai-Hill failure criterion for plane stress multiaxial fatigue loading into a 'general extended Tsai-Hill fatigue failure criterion':

$$M_{11}^2\left(\frac{\sigma_{xx}}{\overline{\sigma}_{11}}\right)^2 + M_{22}^2\left(\frac{\sigma_{xx}}{\overline{\sigma}_{22}}\right)^2 - M_{11}M_{22}\left(\frac{\sigma_{xx}}{\overline{\sigma}_{11}}\right)^2 + M_{12}^2\left(\frac{\sigma_{xx}}{\overline{\sigma}_{12}}\right)^2 = 1$$

(10)

where all in-plane stress components are expressed in terms of σ_{xx} through stress transformations between the structural and material axes coordinate system, the stress ratios R_{xx}, R_{yy} and R_{xy} and the ratios α and β between σ_{xx} and σ_{yy} and σ_{xx} and σ_{xy} respectively. The fatigue strengths $\overline{\sigma}_{ij}$ are functions of number of cycles N, frequency f and stress ratios R_{ij} and are experimentally determined in advance (Jen and Lee (1998a)).

The theory was applied to quasi-isotropic and cross-ply carbon/PEEK laminates, but a larger error for the $[\pm45°]_{4s}$ laminates indicated that further refinements are necessary.

Philippidis and Vassilopoulos (1999) proposed a multiaxial fatigue failure criterion, which is very similar to the well known Tsai-Wu quadratic failure criterion for static loading:

$$F_{ij}\,\sigma_i\,\sigma_j + F_i\,\sigma_i - 1 \leq 0 \qquad i,j = 1,2,6$$

(11)

where F_{ij} and F_i have become functions of the number of cycles N, the stress ratio R and the frequency of loading ν.

The values of the static failure stresses X_t, X_c, Y_t, Y_c and S for the calculation of the tensor components F_{ij} and F_i have further been replaced by the S-N curve values of the material along the same directions and under the same conditions. Although, doing so, five S-N curves are required, the number was reduced to three, when assuming that $X_t = X_c$ and $Y_t = Y_c$.

The researchers preferred to use the laminate properties instead of the lamina properties to predict the laminate behaviour, as they state that this enhances the applicability of the criterion to any stacking sequence of any type of composite (e.g. unidirectional, woven or stitched layers), because the S-N curves for the laminate account for the different damage types occurring in these various types of composite materials.

Philippidis and Vassilopoulos compared their own results against the above-mentioned fatigue failure criterion proposed by Fawaz and Ellyin (1994). They concluded that the criterion by Fawaz and Ellyin was very sensitive to the choice of the reference S-N curve and that the predictions for tension-torsion fatigue of cylindrical specimens were not accurate. Under multiaxial loading the model by Philippidis and Vassilopoulos can produce acceptable fatigue failure loci for all the data considered, but their choice of a multiaxial fatigue strength criterion based on the laminate properties, implies that for each laminate stacking sequence, a new series of experiments is required.

Plumtree and Cheng (1999) indicated that for multiaxial fatigue of metals, the Smith Watson Topper (SWT) parameter appeared to be a valid fatigue parameter. This parameter has the same dimensions as the strain energy density and is defined as the maximum stress times the tensile strain range. A similar definition was now proposed by Plumtree and Cheng for off-axis unidirectional composites:

$$\Delta W^* = \sigma_{22}^{max} \Delta \varepsilon_{22} + \tau_{12}^{max} \Delta \gamma_{12} / 2 \tag{12}$$

where the fatigue parameter ΔW^* accounts for the crack opening modes in off-axis loading: an opening mode normal to the fibres (σ_{22}) and shear parallel to the fibres (τ_{12}).

A best linear fit between the fatigue parameter ΔW^* and the number of reversals to failure $2N_f$ in a log-log coordinate system has been established for unidirectional E-glass/epoxy composites in off-axis fatigue and has then been used to predict fatigue life for other off-axis loading angles.

Bond (1999) has developed a semi-empirical fatigue life prediction methodology for variable-amplitude loading of glass fibre-reinforced composites. The S-N curve is in this case described by the law:

$$\sigma_{max} = b \cdot \log(N) + c \tag{13}$$

where b and c are fourth-order polynomials in function of the ratio range R''. This arbitrary defined function must provide sequential modes of cyclic loading. For the tension-tension regime in the Goodman-diagram for example, R is in the range $0 < R < 1$ and R'' is defined as $R'' = 4 + R$. It is not clear at all how these relations between R and R'' are established in order to develop the fatigue life model.

Other recent investigations to use more complex fatigue models than the traditionally used linear model to characterize the S-N curve can be found in Castillo *et al* (1999) and Revuelta *et al* (2000).

Xiao (1999) has modelled the load frequency effect for thermoplastic carbon/PEEK composites. Fatigue life prediction for 5 Hz and 10 Hz was based on the S-N data at 1 Hz. The reference S-N curve was modelled by a four-parameter power law relation:

$$p = p_0 + \frac{1 - p_0}{(1 + \tau N)^n} \tag{14}$$

where $p = \sigma/\sigma_u$ and $p_0 = \sigma_0/\sigma_u$, in which σ_u is the static strength and σ_0 is the fatigue limit below which stress level no fatigue failure occurs; τ and n are determined by curve fitting.

The S-N curve at 1 Hz was used as the reference curve. Since the maximum temperature during fatigue tests at 1 Hz was about 39 °C in average, 40 °C was chosen to be the reference temperature. It was then assumed that the iso-thermal S-N curves at elevated temperatures (due to hysteretic heating) can be estimated by shifting the reference S-N curve with two shifting factors a_T and b_T. Further an iso-strength plot is needed to model the fatigue life prediction under non-isothermal conditions, as the temperature effect associated with hysteretic heating is non-isothermal. These plots are constructed by drawing a horizontal line in the σ-log(N_f) diagram for a certain stress level, intercepting the iso-thermal S-N curves.

Finally, by calculating the heating rate q from the area of the hysteresis loop, the temperature rise due to hysteretic heating can be calculated. In that way, the correlation between temperature and fatigue testing frequency is established. Fatigue life is finally determined as the intersection point of the temperature curve and the iso-strength curve in a temperature-log(N_f) plot.

Fatigue life was predicted for AS4/PEEK $[\pm 45°]_{4s}$ laminates at 5 Hz and 10 Hz based on an S-N curve generated at 1 Hz. For 10 Hz the measured value was considerably lower than the predicted one, which was possibly due to the delayed heat-transfer.

Miyano *et al* (1994, 2000) developed a model for predicting tensile fatigue life of unidirectional carbon fibre-reinforced composites. The method is based on four hypotheses: (i) same failure mechanisms for constant-strain-rate loading, creep and fatigue failure, (ii) same time-temperature superposition principle for all failure strengths, (iii) linear cumulative damage law for monotonic loading, and (iv) linear dependence of fatigue strength upon stress ratio.

First, a master curve for constant-strain-rate strength and fatigue strength at zero stress ratio was obtained through experimental testing (hypothesis (i) and (ii)). All tests were conducted at temperatures between 50 °C and 150 °C. Fatigue frequencies were in the range 0.02 Hz – 2.0 Hz. Applying hypotheses (i) and (iii), a master

curve for the creep strength was predicted from the master curve of constant-strain-rate strength using the linear cumulative damage law.

It was further supposed that the creep strength can be considered as the fatigue strength at unit stress ratio R = 1 and arbitrary frequency f with the failure time for creep t_c being equal to the failure time for fatigue ($t_f = N_f/f$). Assuming further that the fatigue strength linearly depends upon the stress ratio (hypothesis (iv)), the fatigue strength $\sigma_f(t_f; f, R, T)$ at an arbitrary combination of frequency f, stress ratio R and temperature T was estimated as:

$$\sigma_f(t_f; f, R, T) = \sigma_{f:1}(t_f; f, T) \cdot R + \sigma_{f:0}(t_f; f, T) \cdot (1 - R) \tag{15}$$

where $\sigma_{f:1}(t_f; f, T)$ is the creep strength and $\sigma_{f:0}(t_f; f, T)$ is the fatigue strength at zero stress ratio.

The model was applied to experimental data for carbon/epoxy composite rings, produced by filament winding method. The predictions deviate from the experimental data, when the temperature is above the glass-transition temperature.

Epaarachchi and Clausen (2000) proposed an empirical fatigue law:

$$\frac{d\sigma}{dt} = -a\,\sigma_{max}(1 - R)^\gamma\,t^{-k} \tag{16}$$

where a and k are constants, γ is set fixed to 1.6 (derived from assumptions on fatigue crack propagation rate), σ_{max} is the applied stress level, R is the stress ratio and t is a measure of time. The equation can be rearranged to give:

$$\left(\frac{\sigma_{ult}}{\sigma_{max}} - 1\right)\frac{1}{(1 - R)^\gamma}\,f^\beta = \alpha(N^\beta - 1) \tag{17}$$

where f is the frequency, γ is set to 1.6, N is the number of cycles to failure, f is the loading frequency and α and β are constants. Since the right hand side of the equation is constant for a given σ_{max}, regardless of the value of f and R, the parameters α and β can be determined experimentally. The model was applied to fatigue data from literature for glass/epoxy and glass/polypropylene specimens.

3.2   Phenomenological models to predict residual stiffness/strength

3.2.1   Residual stiffness models

Residual stiffness models describe the degradation of the elastic properties during fatigue loading. To describe stiffness loss, the variable D is often used, which in the one-dimensional case is defined through the well-known relation $D = 1 - \dfrac{E}{E_0}$, where E_0 is the undamaged modulus. Although D is often referred to as a damage variable, the models are classified as phenomenological models and not as progressive damage models, when the damage growth rate dD/dN is expressed in terms of macroscopically observable properties, and is not based on the actual damage mechanisms.

Hwang and Han (1986a, 1986b) introduced the concept of the 'fatigue modulus', which is defined as the slope of applied stress and resultant strain at a specific cycle. The fatigue modulus degradation rate is assumed to follow a power function of the number of fatigue cycles:

$$\frac{dF}{dn} = -A\,c\,n^{c-1} \tag{18}$$

where A and c are material constants. Further they assumed that applied stress σ_a varies linearly with resultant strain in any arbitrary loading cycle, so that:

$$\sigma_a = F(n_i) \cdot \varepsilon(n_i) \tag{19}$$

where $F(n_i)$ and $\varepsilon(n_i)$ are the fatigue modulus and strain at loading cycle n_i, respectively. After integration and introducing the strain failure criterion, the fatigue life N can be calculated as:

$$N = \left[B(1-r) \right]^{1/c} \tag{20}$$

where $r = \dfrac{\sigma_a}{\sigma_u}$ is the ratio of the applied cyclic stress to the ultimate static stress, B and c are material constants.

Hwang and Han (1986a) proposed three cumulative damage models based on the fatigue modulus F(n) and the resultant strain. The presented model III shows better agreement with experimental data than the first models I and II. It is proposed as:

$$D = \frac{r}{1-r} \cdot \left[\frac{F_0}{F(n)} - 1 \right] \tag{21}$$

Failure occurs when:

$$D = \sum_{i=1}^{m} \Delta D_i = 1 \tag{22}$$

where ΔD_i is the amount of damage accumulation during fatigue at stress level r_i and m is the number of load sequences until final failure.

As explained in their work on the fatigue modulus concept (Hwang and Han (1986b)), the cumulative damage model can be expressed as a function of the number of cycles too, but the researchers advise to define the cumulative damage model by physical variables rather than by number of cycles for a better understanding of the multi-stress level fatigue phenomena.

The cumulative damage models proposed by Hwang and Han have been used by Kam *et al* (1997, 1998) to study the fatigue reliability of graphite/epoxy composite laminates under uniaxial spectrum stress using the modified β-method.

A recent review of cumulative damage models for *homogeneous* materials (more specifically metals and their alloys) is given by Fatemi and Yang (1998). Some of these models have been applied to fibre-reinforced composites as well.

Sidoroff and Subagio (1987) proposed the following model for the damage growth rate:

$$\frac{dD}{dN} = \begin{vmatrix} \dfrac{A.(\Delta\varepsilon)^c}{(1-D)^b} & \text{in tension} \\ 0 & \text{in compression} \end{vmatrix} \tag{23}$$

where the variable $D = 1 - \dfrac{E}{E_0}$; A, b and c are three material constants to be identified from experiments and $\Delta\varepsilon$ is the applied strain amplitude.

The model was applied to the results from three-point bending tests on glass-epoxy unidirectional composites under fixed load amplitudes.

Van Paepegem and Degrieck (2000b, 2001) have implemented the model of Sidoroff and Subagio into a commercial finite element code. Each Gauss-point was assigned a state variable D, which is related with longitudinal stiffness loss. After calculating one fatigue loading cycle (with the possibility to include inertia and damping forces, contact conditions, friction,…), the procedure loops over all Gauss-points and makes an estimate of the value of the local 'cycle jump'; this is the number of cycles that can be jumped over without loss of accuracy on the integration of the fatigue evolution law dD/dN for that particular Gauss-point. Finally, the

global 'cycle jump' for the whole finite element mesh is defined as a certain fractile of the cumulative relative frequency distribution of all local 'cycle jump' values. The damage state of the simulated cycle is then extrapolated over the number of cycles that equals the value of the global 'cycle jump', after which another fatigue loading cycle is again fully calculated.

The finite element implementation was used to simulate the fatigue behaviour of glass fabric/epoxy specimens, which were fatigue loaded as a cantilever beam in displacement-control. Due to the different damage distribution through the thickness and along the specimen length, stresses were continuously redistributed during fatigue life. This was accurately simulated by the finite element implementation.

The model of Sidoroff and Subagio has been adopted very recently by other researchers, but often in terms of stress amplitude instead of strain amplitude.

Vieillevigne *et al* (1997) defined the damage growth rate as:

$$\frac{dD}{dN} = K_d \frac{\sigma^m}{(1-D)^n} \tag{24}$$

where σ is the local applied stress, m and n are fixed parameters, while K_d depends on the dispersion. In compression regime, dD/dN was again assumed to be zero. The formula was applied to three-point bending tests.

Kawai (1999) modified the model for off-axis fatigue of unidirectional carbon fibre-reinforced composites:

$$\frac{d\omega}{dN} = \frac{K\left(\sigma^*_{max}\right)^n}{(1-\omega)^k} \tag{25}$$

where K, n and k are material constants and σ^*_{max} is a non-dimensional effective stress corresponding to a maximum fatigue stress and is defined as:

$$\sigma^*_{max} = Max\left\{\sqrt{\left(\frac{\sigma_{11}}{X}\right)^2 - \frac{\sigma_{11}\sigma_{22}}{X^2} + \left(\frac{\sigma_{22}}{Y}\right)^2 + \left(\frac{\tau_{12}}{S}\right)^2}\right\} \tag{26}$$

where X, Y and S are the static tensile strength, transverse strength and shear strength, respectively.

Whitworth (1987) proposed a residual stiffness model for graphite/epoxy composites:

$$\left(\frac{E(N^*)}{E(0)}\right)^a = 1 - H \cdot \left(1 - \frac{S}{R(0)}\right)^a N^* \tag{27}$$

where $N^* = n/N$ is the ratio of applied cycles to the fatigue life N, S is the applied stress level, R(0) is the static strength, E(0) is the initial modulus, and a and H are parameters which are independent of the applied stress level.

This residual stiffness model was used by Whitworth (1990) to propose a cumulative damage model, where the damage function has been defined as:

$$D = \left[\frac{H \cdot (1-\bar{S})^a}{1-\bar{S}^a}\right] \cdot \frac{n}{N} \tag{28}$$

where $\bar{S} = \dfrac{S}{R(0)}$ is the normalized applied stress range and a and H are the parameters. When D = 0, no cycles have been applied and E = E(0). When D = 1, then the residual modulus equals the failure stiffness E_f.

This damage model has been extended to predict the remaining life of composite specimens subjected to variable amplitude fatigue loading. To determine the fatigue failure criterion in case of variable amplitude loading,

Whitworth used the 'equivalent cycles approach'. In this approach, the number of cycles at a particular stress condition in a variable amplitude loading group is transformed into an equivalent number of cycles at some reference stress condition such that the original and transformed groups produce the same damage. When the sum of the damage values at each stress level reaches one, failure occurs. Such an approach holds the assumption that the behaviour of the composite specimen is history independent. The model was tested on the experimental data for two-stress level fatigue loading.

Recently, Whitworth (1998) proposed a new residual stiffness model, which follows the degradation law:

$$\frac{dE^*(n)}{dn} = \frac{-a}{(n+1)\left[E^*(n)\right]^{m-1}} \tag{29}$$

where $E^*(n) = E(n)/E(N)$ is the ratio of the residual stiffness to the failure stiffness $E(N)$, n is the number of loading cycles and a and m are parameters that depend on the applied stress, loading frequency,… By introducing the strain failure criterion, the residual stiffness $E(n)$ can be expressed in terms of the static tensile strength S_u and a statistical distribution of the residual stiffness can then be obtained, assuming that the static ultimate strength can be represented by a two-parameter Weibull distribution.

Yang *et al* (1990) have developed a residual stiffness model for fibre-dominated composite laminates:

$$\frac{dE(n)}{dn} = -E(0)Q\nu n^{\nu-1} \tag{30}$$

where Q and ν are two parameters which are correlated by a linear equation. Experimental data revealed that ν could be written as a linear function of the applied stress level. The researchers have also derived a statistical distribution of the residual stiffness.

They observed that this model was not immediately applicable to matrix-dominated composite laminates, because then the stress-strain curve is no longer linear. Yang *et al* (1992) have extended the model for matrix-dominated composites by replacing the modulus $E(n)$ by the fatigue modulus $F(n)$. The latter is defined as the applied stress level S divided by the corresponding strain at the n-th cycle. Through the modelling of the non-linear stress-strain response, they derived an expression, relating the fatigue modulus $F(0)$ with the initial stiffness $E(0)$. They have proved that this new damage law is a particular case of the above-mentioned damage model for fibre-dominated composites. The model for matrix-dominated behaviour was applied to the fatigue behaviour of $[\pm45°]_{2S}$ graphite/epoxy laminates.

Lee *et al* (1996) used their model (30) to predict failure stiffness and fatigue life for composite laminates subjected to service loading spectra. An empirical criterion for the fatigue failure strain $\varepsilon(N)$ was proposed. Since the experimental results showed large scatter in the third region of stiffness reduction, the researchers proposed to consider the fatigue failure strain at the end of the secondary region.

Hansen (1997, 1999) developed a fatigue damage model for impact-damaged woven fabric laminates, subjected to tension-tension fatigue:

$$\beta = A \int_0^N \left(\frac{\varepsilon_e}{\varepsilon_0}\right)^n dN \qquad \beta \leq \beta_{lim} \tag{31}$$

where N is the number of cycles, ε_e is the effective strain level and ε_0 the reference strain level, A and n are constants. The damage variable β is related to the elastic properties by the relations:

$$E = E_0(1-\beta)$$
$$\nu = \nu_0(1-\beta) \tag{32}$$

The experiments revealed that the tension-tension fatigue behaviour of the woven composites was affected by the low-energy impact damage for high-cycle fatigue (moderate or low peak load levels), but not for low-cycle

fatigue (high peak load levels). Infrared thermography, which monitors the heating by internal losses and friction within the damaged regions, was found to be very successful in detecting damage initiation and growth.

Brøndsted *et al* (1997a, 1997b) extended stiffness reduction to the life time prediction of glass fibre-reinforced composites. The predictions are based on experimental observations from wind turbine materials subjected to constant amplitude loading, variable amplitude block loading and stochastic spectrum loading. The material is a four-layer 90:10 fabric with a chopped strand mat on both sides.
The stiffness change is calculated as:

$$\frac{d\left(\dfrac{E}{E_1}\right)}{dN} = -K \cdot \left(\frac{\sigma}{E_0}\right)^n \tag{33}$$

where E is the cyclic modulus after N cycles, E_1 is the initial cyclic modulus, E_0 is the static modulus, σ is the maximum stress and K is a constant. This expression is based on their observed relationship between the stiffness and fatigue cycles in the second stage of the stiffness degradation curve:

$$\frac{E}{E_1} = A \cdot N + B \tag{34}$$

where the stress dependence of the parameter A is assumed to be a power law relationship.
The researchers supposed that the stiffness change is history independent. The model can then be utilized to predict the lifetime for variable amplitude loading conditions.

3.2.2   Residual strength models

Two types of residual strength models can be distinguished: the *sudden death* model and the *wearout* model.
When composite specimens are subjected to a high level state of stress (low-cycle fatigue), the residual strength as a function of number of cycles is initially nearly constant and it decreases drastically when the number of cycles to failure is being reached. The *sudden death* model (Chou and Croman (1978, 1979)) is a suitable technique to describe this behaviour and is especially used for high-strength unidirectional composites.
However at lower level states of stress, the residual strength of the laminate, as a function of number of cycles, degrades more gradually. This behaviour is described by degradation models which are often referred to as *wearout* models. These models generally incorporate the 'strength-life equal rank assumption' which states that the strongest specimen has either the longest fatigue life or the highest residual strength at runout. This assumption has been experimentally proved by Hahn and Kim (1975). It should be noted that this assumption may not hold if competing failure modes are observed during the fatigue tests (Sendeckyj (1981)).
Whether or not the residual strength models mentioned below, are applicable to both low- and high-cycle fatigue, can not always be determined. Most researchers do not provide experimental results in both ranges of cycles.

In the *wearout* model, which was initially presented by Halpin *et al* (1973), it is assumed that the residual strength R(n) is a monotonically decreasing function of the number of cycles n, and that the change of the residual strength can be approximated by a power-law growth equation:

$$\frac{dR(n)}{dn} = \frac{-A(\sigma)}{m[R(n)]^{m-1}} \tag{35}$$

where $A(\sigma)$ is a function of the maximum cyclic stress σ, and m is a constant.
This procedure was followed by a lot of researchers afterwards (Hahn and Kim (1975, 1976), Chou and Croman (1978, 1979), Yang (1978)). The survey by Kedward and Beaumont (1992) has given an overview of the use of *wearout* models in certification methodologies.

In the work of Yang and Jones (1981) the following form for the residual strength curve has been proposed:

$$R^v(n) = R^v(0) - \frac{R^v(0) - \sigma^v}{R^c(0) - \sigma^c} K S^b n \tag{36}$$

where R is the residual strength, n is the number of cycles, σ is the maximum cyclic stress, v is a parameter, $c = \alpha/\alpha_f$ is the ratio of the shape parameter of the ultimate strength to that of the fatigue life and $\tilde{N} = 1/KS^b$ is the S-N curve of the characteristic fatigue life, where K and b are constants and S is the stress range.

Yang and Jones also derived expressions for the distributions of fatigue life and residual strength under dual stress levels and spectrum loadings in terms of three-parameter and two-parameter Weibull distributions respectively. Using these expressions, the load sequence effects were investigated for dual stress fatigue loadings and spectrum loadings.

Daniel and Charewicz (1986) studied damage accumulation in cross-ply graphite/epoxy laminates under cyclic tensile loading. They proposed a model based on the normalized change in residual strength:

$$\left[\frac{1 - f_r}{1 - s} \right] = g\left(\frac{n}{N} \right) \tag{37}$$

where $f_r = \dfrac{F_r}{F_0}$ is the normalized residual strength, $s = \dfrac{\sigma_a}{F_0}$ is the normalized applied cyclic stress, N is the number of cycles to failure at stress σ_a and g(n/N) is a function of the normalized number of cycles which however has not been determined in their article. The researchers mentioned that the model is far from satisfactory, because it completely relies on a good definition of the residual strength curve. The experimental determination of the residual strength curve, which obviously is not a single-valued function of the number of cycles, is very difficult in view of the considerable scatter of the experimental data.

Further Daniel and Charewicz assume that the fatigue damage is only a function of the residual strength, such that a specimen cycled at a stress σ_1 for n_1 cycles has the same damage as a specimen cycled at a stress σ_2 for n_2 cycles, if they have the same residual strength F_r after their respective cycles n_1 and n_2. This definition allows the determination of equal damage curves in the (σ, n) plane. Residual life predictions thus can be made once the equal damage curves are determined.

According to Rotem (1986), the initial static strength is maintained almost up to final failure by fatigue. He then defined an imaginary strength S_0 in the first loading cycle, which has a higher value than the static strength. If the S-N curve for tension-tension fatigue of graphite/epoxy laminates is expressed as:

$$s = 1 + K \cdot \log(N) \tag{38}$$

where $s = \dfrac{S_f}{S_0}$ with S_f the fatigue strength for constant amplitude and S_0 the imaginary strength, then the remaining fatigue life after a certain amount of load cycles can be given by a curve similar to the S-N curve, but with a different slope and passing through the point S_0. Such a curve is called a 'damage line' and a family of such damage lines is defined by:

$$s = 1 + k \cdot \log(N) \quad k < K \tag{39}$$

As long as the degradation of the residual strength is situated in the region between the imaginary strength and the actual static strength, there is no apparent degradation of the strength.

The cumulative fatigue theory based on these assumptions, was extended by Rotem (1991) to predict the S-N curve of a composite laminate which is subjected to an arbitrary, but constant stress ratio R.

Extensive experimental and theoretical research has been done by Schaff and Davidson (1997a, 1997b). They presented a strength-based wearout model for predicting the residual strength and life of composite structures subjected to spectrum fatigue loading.

The following model for the residual strength was proposed:

$$R(n) = R_0 - (R_0 - S_p)\left(\frac{n}{N}\right)^{\nu}$$

(40)

where R is the residual strength, S_p is the peak stress magnitude of the loading and ν is a parameter. Linear strength degradation corresponds to $\nu = 1$. Sudden death behaviour is obtained for $\nu \gg 1$, and a rapid initial loss in strength is obtained for $\nu < 1$.

This model was applied first to two-stress amplitude fatigue loading. Because the decrease of strength under stress level S_2 depends on the number of cycles n_1 that the material has previously sustained under the stress level S_1, the contribution of (S_1, n_1) has to be considered. Therefore an effective number of cycles n_{eff} has been defined, such that (S_1, n_1) causes the same decrease of strength as (S_2, n_{eff}).

Schaff and Davidson also investigated the importance of the 'cycle mix effect': laminates that experience small cycle blocks of higher stress, have reduced average fatigue lives as compared to laminates that are subjected to large cycle blocks of higher stress, although the total number of cycles they have been subjected to that higher stress, is the same for both laminates at the end of the experiment.

This effect is particularly important in the second part of their study (Schaff and Davidson (1997b)), where experimental results from so-called FALSTAFF spectrum loadings are used. FALSTAFF stands for 'Fighter Aircraft Loading STAndard For Fatigue' and is a standardized random-ordered loading spectrum that simulates the in-flight load-time history of fighter aircraft. The 'cycle mix effect' is important in the FALSTAFF spectrum, as many of the constant amplitude segments are only a few cycles in length. The effect is accounted for in the model through the application of a 'cycle mix factor', which is applied only when the magnitude of the mean stress increases from one loading segment to the next.

The model shows good correlation to a variety of experimental results, including the complex FALSTAFF loading.

Caprino and D'Amore (1998) conducted fatigue experiments in four-point bending on a random continuous-fibre-reinforced thermoplastic composite.

The hypothesis for their damage law is that the residual strength undergoes a continuous decay, following a power law:

$$\frac{d\sigma_n}{dn} = -a_0 \cdot \Delta\sigma \cdot n^{-b}$$

(41)

where σ_n is the residual strength after n cycles, $\Delta\sigma = \sigma_{max} - \sigma_{min}$ is a measure for the influence of the stress ratio R, and a_0 and b are two constants.

Caprino and D'Amore stressed the fact that a reliable model should reflect both the influence of the stress ratio R and the different fatigue behaviour at low- and high-cycle fatigue. Indeed there appeared to be a transition in failure mode from matrix shear yielding at low-cycle fatigue (high stress levels) to a single crack growth at high-cycle fatigue (low stress levels) for the studied material.

Moreover Caprino *et al* (1998) observed that the higher the material sensitivity to stress amplitude, the lower its sensitivity to the number of cycles. This implies that comparing different materials on the basis of their fatigue response at low-cycle fatigue does not necessarily leads to the same conclusions for high-cycle fatigue.

Recently, Caprino and Giorleo (1999) have applied their model to four-point bending fatigue of glass-fabric/epoxy composites, while Caprino (2000) used the residual strength model for tension-tension fatigue of carbon fibre-reinforced composites. In the case of the carbon fibre-reinforced composites, Caprino concluded that the model can predict the fatigue life, but that the experimentally measured residual strength does not follow the path as described by the residual strength law (41). Therefore, in that case, the model must be considered as a fatigue life model and not as a residual strength model.

Whitworth (2000) used a previously proposed residual stiffness model (Whitworth (1998), see Equation (29)) to evaluate the residual strength degradation. Thereto the failure stiffness E(N) in Equation (29) is determined by introducing the strain failure criterion:

$$\frac{S}{S_U} = c_1 \left[\frac{E(N)}{E(0)} \right]^{c_2}$$

(42)

where S is the applied stress level, S_U is the ultimate strength, E(0) is the initial stiffness and E(N) is the failure stiffness. The parameters c_1 and c_2 were introduced to account for non-linear effects. Finally the residual strength can be expressed as:

$$S_R^\gamma = S_U^\gamma - \frac{n}{N} \left(S_U^\gamma - S^\gamma \right)$$

(43)

where S_R is the residual strength and γ is a parameter. The fatigue life N in the equation (43) can now be expressed in terms of ultimate strength S_U and applied stress level S, based on the evolution law for the residual stiffness degradation.

Yao and Himmel (2000) assumed that the residual strength behaviour under tension fatigue for fibre-reinforced polymers can be described by the function:

$$R(i) = R(0) - [R(0) - S] \frac{\sin(\beta x) \cos(\beta - \alpha)}{\sin \beta \cos(\beta x - \alpha)}$$

(44)

where R(i) is the residual strength at the i-th loading cycle, R(0) is the static strength, S is the stress loading level, $x = i/N_f$ and α and β are parameters to be determined through experiments. For specimens which fail under compressive loading, the residual strength was assumed to obey the degradation law:

$$R(i) = R(0) - [R(0) - S] \left(\frac{i}{N_f} \right)^\nu$$

(45)

where ν is a strength degradation parameter depending on the stress ratio and the peak stress.
Then the cumulative damage was assessed according to the assumption that the damage state can be treated equivalently if the residual strengths are equal.
The theory was applied to block loading experiments for glass/epoxy cross-ply laminates and carbon/epoxy composites.

3.3 Progressive damage models

Progressive damage models differ from the above mentioned models in that they introduce one or more properly chosen damage variables which describe the deterioration of the composite component. These models are based on a physically sound modelling of the underlying damage mechanisms, which lead to the macroscopically observable degradation of the mechanical properties. The models have been subdivided into two classes: the damage models which predict the damage growth as such (e.g. number of transverse matrix cracks per unit length, size of the delaminated area), and the models which correlate the damage growth with the residual mechanical properties (stiffness/strength).

3.3.1 Progressive damage models predicting damage growth

Some models have been proposed to model damage accumulation for specific damage types, such as matrix cracks and delaminations. Several models classified in this category, make use of experiments on notched specimens to initiate a specific damage type at a well-known site.

Owen and Bishop (1974) were among the first researchers to investigate a wide range of glass fibre-reinforced composites. They tried to predict the initiation of damage at central holes in the specimens under static and fatigue loading.

They concluded that there exists a substantial adverse size effect for some types of materials used, because damage and failure in the large specimens (width and diameter of the central hole ten times larger and the same length-to-width ratio) occurred at between 48% and 73% of the corresponding values for the small specimens.

Moreover central-holed specimens with one slit at each side of the hole were subjected to fatigue loading. Because the measurement of the crack lengths was experimentally difficult to do, Owen and Bishop related crack length to the specimen compliance.

They concluded that the Paris power relationship is applicable to the fatigue crack growth rate in the two glass fibre-reinforced materials examined.

Biner and Yuhas (1989) investigated the growth of short fatigue cracks at notches in woven glass/epoxy composites. It was demonstrated that initiation and growth rate of short cracks emanating from blunt notches can be accurately described by an effective stress intensity factor range ΔK_{eff}. For short cracks, the value of ΔK_{eff} was calculated for a notch plus crack geometry using conformal mapping techniques. When the crack lengths were sufficiently long, ΔK_{eff} converged to ΔK.

Bergmann and Prinz (1989) and Prinz (1990) proposed a specific model for delamination growth:

$$\frac{dA_i}{dN} = \hat{c} \cdot f(G_{it})^n \qquad (46)$$

where A_i is the delaminated area, G_{it} is the maximum amplitude of the energy release rate and $\hat{c}$ and n are experimentally determined values.

For experimental purposes, graphite/epoxy specimens of stacking order $[0_2,+45,0_2,-45,0,90]_S$ with a central hole and unidirectional specimens of stacking order $[0_n, 0_m]_S$ with *2m* severed central plies $[0_m]_S$ were subjected to fatigue loadings. In case of the latter configuration, an artificial cut separated the *2m* central plies. As a consequence a lateral crack occurred in the course of cyclic loading in the area of the artificial separations, from which two partial delaminations developed at each of the interfaces of the separated and the continuous plies.

The buckling of delaminated sections was studied as well and an estimate of the critical buckling load F_c was determined.

Dahlen and Springer (1994) proposed a semi-empirical model for estimating delamination growth in graphite/epoxy laminates under cyclic loading, including mode I, mode II and mixed-mode conditions. It was assumed that mode III does not significantly contribute to the delamination growth, and that viscoelastic and thermal effects are negligible. Then the crack growth rate is described, using dimensionless grouping of the involved variables and a Paris similar growth law:

$$\Delta a \frac{\sigma_f^2}{E_y G_{crit}} = A \left(U \frac{G_{max}}{G_{crit}} \right)^b \qquad (47)$$

where Δa is the delamination growth normal to the circumference of the existing delamination, σ_f is the ply strength, E_y is the transverse ply modulus, G_{crit} is the critical energy release rate with contributions from mode I and mode II, A and b are parameters which depend on the material and the relative contributions of mode I and mode II to the delamination growth, U is a function of G_{max}/G_{crit} and of G_{min}/G_{max}. The form of this function U depends on whether or not there is a shear reversal during mode II conditions, because the delamination grows at different rates in the absence or presence of shear reversal. G_{max} is the total maximum energy release rate during the cycle under consideration.

Three types of tests were performed on graphite/epoxy specimens: (i) mode I and mode II delamination growth under static loads, (ii) mode I and mode II delamination growth under cyclic loads, (iii) mixed mode delamination growth under cyclic loads. Thereto, double cantilever beam, end notched cantilever beam, mixed mode end notched cantilever beam and mixed mode bending tests were performed.

Xiao and Bathias (1994a, 1994b) studied notched and unnotched woven glass/epoxy laminates with a strong unbalanced character: the mechanical properties in the 'warp' direction were much higher than those in the 'weft' direction. Although they did not propose a fatigue evolution law, they introduced fatigue ratios to compare the experimental data. The results showed that the unnotched and notched laminates have the same ratios of the fatigue strength to the ultimate tensile strength and that the fatigue strength ratios of notched and unnotched laminates for the three stacking sequences considered, are respectively equal to their respective static strength ratios.

They also reported that the stacking sequence influences the fatigue life: when 90° layers are constrained by 0° layers, the damage in the 90° layers cannot easily cross the interface between the 90° plies and the other plies. As a consequence the damage trace is very sinuous through the thickness.

Feng *et al* (1997) developed a model for predicting fatigue damage growth in carbon fibre-reinforced specimens due to matrix cracking. From experimental observations, it was concluded that the mode I crack growth could be described by a modified Paris law:

$$\frac{dA}{dN} = D\, G_{max}^{n}$$
(48)

where A is the damage area due to matrix cracking, N is the number of fatigue cycles, G_{max} is the maximum strain-energy release rate in a fatigue cycle, and D and n are material constants. Through finite element calculations, the value of G_{max} is evaluated and the finite element analysis of the local region is then run many times to simulate damage growth. This procedure allows to define the mathematical relationship between A and G_{max}.

When the fibre strain exceeds the fibre fracture strain, fatigue failure occurs. In that way, a prediction of the fatigue life N_f and the final value of the damage area A_f can be made.

The damage area due to matrix cracking was predicted for a notched I-beam subjected to four-point fatigue bending, and for a notched coupon subjected to tensile fatigue loading.

Hénaff-Gardin *et al* (1997, 2000) have studied progressive matrix cracking in cross-ply laminates. The propagation law under fatigue was established as:

$$\frac{dS}{dN} = A\left(\frac{G_I}{G_{max}}\right)^{n}$$
(49)

where S is the crack surface, G_I is the strain energy release rate for the current crack density, G_{max} is the value of the strain energy release rate when the first matrix crack initiates, and A and n are constants, which are determined from experimental measurements of crack density. When G_{max} is lower than G_{Ic}, the initiation of the first matrix crack requires a micro-damage accumulation during the first fatigue cycles. In that case, a phenomenological law was used to predict the cycle number, necessary for transverse cracking initiation.

The same research group (Gamby *et al* (1997)) studied how matrix cracks start from a free edge and propagate towards the centre of specimens with three sorts of stacking sequences: $[0_2,+45,-45,0_2]_s$, $[+45_2,-45_2]_s$ and $[0_4,45_2]_s$. The first damage mode consisted of matrix cracks in the ±45° plies and it appeared that the crack density in each ply was roughly constant along a line parallel to the specimen axis: it only depended on the distance x from a free edge and the number of cycles N. They postulated a principle of conservation of the number of crack tips in a suitable volume, which gave rise to a nonlinear wave equation governing crack density as a function of both number of cycles N and distance x from a free edge of the specimen.

Bartley-Cho *et al* (1998) studied ply cracking behaviour of quasi-isotropic graphite/epoxy laminates under constant-amplitude tension-tension and tension-compression fatigue. Block loading tests were also performed in tension-tension and tension-compression to assess the load sequence effect on ply cracking. The authors outlined the following approach to calculate ply cracking in fatigue.

First, for a given laminate stress σ_{lam}, the corresponding ply stresses are calculated. Two different lay-ups were used: $[0/\pm45/90]_{s3}$ and $[0/\pm45/90]_{s4}$. It was observed that the cracks initiate in the 90° plies before they initiate in the -45° plies. Due to the stiffer 0° ply being next to the +45° ply, the results show a lag in the +45° ply cracking. A failure function is assumed to predict ply cracking:

$$f = \frac{\sigma_{yy}^2}{YY'} + \left(\frac{1}{Y} - \frac{1}{Y'}\right)\sigma_{yy} + \left(\frac{\sigma_{xy}}{S}\right)^2 = 0.981\,N^{-0.134} \tag{50}$$

where the failure function varies with the number of cycles following an experimentally determined relationship. Y and Y' are the lamina transverse tensile and compressive strengths, respectively, and S is the shear strength.

For the 90° plies, the number of cycles N_i to crack initiation is calculated from Equation (50). Then the number of cycles is increased and the crack density in the 90° plies is calculated from the expression for the average ply crack densities ρ_{crack} which were fitted using the following equation:

$$\rho_{crack} = \rho_\infty\left(1 - \exp\left(-b\log\frac{N}{N_i}\right)\right) \tag{51}$$

where N_i is the number of cycles to crack initiation, b is a constant and ρ_∞ is the saturation crack density. These functions were fitted for different load levels for the 90°, -45° and +45° plies. The researchers observed that, in the absence of another competing damage mode such as delamination, the saturation crack density ρ_∞ increased with applied load level, which is contrary to the general belief that saturation crack density is independent of load-history.

Then, due to cracking in the 90° plies, stresses are redistributed and the reduced effective ply stiffnesses C_{ij} are given by:

$$C_{ij} = C_{ij}^0 \exp(-\alpha_{ij}\,\rho_{crack}t_{ply}) \tag{52}$$

where α_{ij} are experimentally determined stiffness reduction constants and t_{ply} is the ply thickness. These new stresses, which are in fact function of ρ_{crack}, are now used for evaluating the failure function. Then the calculation is proceeded until the number of cycles to crack initiation in the -45° plies is reached, according to the above-mentioned failure function. Finally, both 90° and –45° plies fail.

The model was also applied to block loading experiments revealing a sequence effect where the low/high load sequence resulted in more ply cracks than the high/low load sequence. However since the model is not dependent on load-history, the sequence effect can be predicted neither for tension-tension nor tension-compression block fatigue loading.

Bucinell (1998) developed a stochastic model for the growth of free edge delaminations in composite laminates. The experiments were conducted for the stacking sequence $[\pm45°/90°/0°]_S$ of AS4/3501-6 coupons, where the location of the free edge delamination was observed to appear always in the 45°/90° interface.

The growth model was derived from fracture mechanics principles:

$$\frac{da}{dN} = \alpha\left[\frac{G(\sigma_m,a)}{G_c}\right]^\rho \tag{53}$$

where a is the delamination width, N is the number of cycles, σ_m is the applied load, G is the strain energy release rate and G_c its critical value. The parameters α and ρ are estimated using the delamination width versus number of cycles data for various fatigue load levels.

The researchers reported that further investigation is required to extend the model for delaminations in laminates with other geometries than the geometries used in their research work.

Schön (2000) proposed a simplified method to describe delamination growth in fibre-reinforced composites. The delamination growth rate under fatigue loading is assumed to be described by the Paris law:

$$\frac{da}{dN} = D(\Delta G)^n \tag{54}$$

where a is the delamination crack length, ΔG is the range of the energy release rate, and D and n are constants. In a log-log diagram the law is represented by a straight line and thus only two points are needed to determine the constants D and n.

In order to avoid any confusion when comparing delamination growth results from specimens where tension and compression loading is well defined, with those of simple test specimens, such as the Double Cantilever Beam and End Notched Flexure test specimens, a new parameter Q is introduced and is defined as:

$$\begin{aligned} Q &= R \quad \text{when} \quad -1 \le R \le 1 \\ Q &= \frac{1}{R} \quad \text{when} \quad |R| > 1 \end{aligned} \tag{55}$$

where R is the stress ratio. Further, for mode II loading of the crack, it is possible to have shear reversal and negative Q-values. Instead of ΔG, a quantity ΔG_r, which is the rate of change in energy release rate, is introduced to handle shear reversal and is defined as:

$$\begin{aligned} \Delta G_r &= G_{max} - G_{min} \quad \text{when} \quad Q > 0 \\ \Delta G_r &= G_{max} + G_{min} \quad \text{when} \quad Q < 0 \end{aligned} \tag{56}$$

with G_{max} and G_{min} being the maximum and minimum energy release rates of the delamination crack during a fatigue cycle.

Then two conditions are defined in order to determine D and n:

$$\begin{aligned} \left(\frac{da}{dN}\right) &= \left(\frac{da}{dN}\right)_{th} \quad \text{when} \quad \Delta G_r = \Delta G_{r,th} \\ \left(\frac{da}{dN}\right) &= \left(\frac{da}{dN}\right)_c \quad \text{when} \quad G_{max} = G_{max,c} \end{aligned} \tag{57}$$

The first condition holds for the fatigue delamination growth threshold, which experimentally has been found to occur for a constant change in energy release rate, thus for a value $\Delta G_{r,th}$ being independent of mode ratio and the Q-value. The second condition holds for static fracture, where it is assumed that the maximum energy release rate $G_{max,c}$ during fatigue loading can not be larger than the critical energy release rate G_c from quasi-static loading.

The validity of the model was checked against experimental observations reported in the open literature. The model predicts the n-value to decrease with decreasing Q-value and increasing amount of mode II fracture.

3.3.2   *Progressive damage models predicting residual mechanical properties*

This category of progressive damage models relates the damage variable(s) with the residual mechanical properties (stiffness/strength) of the laminate. The damage growth rate equations are often based on damage mechanics, thermodynamics, micromechanical failure criteria or specific damage characteristics (crack spacing, delamination area,…).

One of the first methods to calculate stiffness reduction due to matrix cracking is the shear-lag model, established by Highsmith and Reifsnider (1982). Through careful examination of edge replicas of crack patterns in specimens of various composite materials, they observed that shear deformations in any given ply were restricted to a thin region in the vicinity of the interfaces of that ply with adjacent plies. Further this region tended to be resin-rich, and thus was less stiff in response to shear loads than the central portion of the lamina. Transverse cracks did extend up to this region, but usually did not extend into it. Hence this thin layer was referred to as the shear layer, in the vicinity of the layer interface. Tensile stresses in the uncracked layers were transferred to the cracked layers via the shear layer.

Based on these observations, a one-dimensional equilibrium equation for a cracked ply in the laminate can then be established. The assembly of the governing equations for the plies within a laminate yields a system of coupled, second order differential equations in displacements. The main problem is to define the thickness of the shear layer.

More recent models to calculate stiffness reduction due to matrix cracking are based on a variational approach, finite element analysis,… References can be found in Nuismer and Tan (1988), Brillaud and El Mahi (1991), El Mahi *et al* (1995), Joffe and Varna (1999), Pradhan *et al* (1999), Smith and Ogin (1999) and Kashtalyan and Soutis (2000).

Reifsnider (1986) further proposed a modelling philosophy to counteract the problem of interaction of damage types. On the one hand it is computationally impossible to account for all of the interactions of all damage types on the microstructural level. On the other hand homogenization of constitutive properties and averaging out the influence of all damage events results in the loss of critical information (Reifsnider (1987)). Therefore Reifsnider (1986) proposed the approach of the 'representative volume concept'. This representative volume is further divided into critical and subcritical elements. In the subcritical elements damage initiation and propagation is modelled on a micromechanical level and the local stress fields are calculated. The other details which are not important for the determination of the local stress field associated with the final failure event are grouped into continuum representations of the critical elements in the representative volume.

In this approach the reduction in strength can be calculated using the integral formulation (Reifsnider (1986)):

$$\frac{S_L^r(\tau_1)}{S_{Lu}^i} = \left[\frac{(F_e/F_L)^i}{F_e(\tau)/F_L(\tau)}\right]_{rv} \cdot \left[1 - \int_0^{\tau_1}(1 - F_e(\tau))k(t)^{k-1}\,d\tau\right] \tag{58}$$

The quantity on the left is the residual strength, normalized by the initial ultimate strength for that failure mode. The first factor on the right is the ratio of the initial stress concentration to the current stress concentration in a representative volume (subscript 'rv'), where F is a generalized failure function. The subscript *e* indicates that the value of that failure function F is evaluated in the critical element, and the subscript *L* indicates the value of F in the laminate at some position remote from the location of the failure event. All quantities in the integral are evaluated in the critical element. The failure function is written as a function of time since the stress state in a critical element changes as damage develops in the subcritical elements around it.

Recent applications of this methodology can be found in Diao *et al* (1995), Subramanian *et al* (1995) and Halverson *et al* (1997).

Talreja (1986, 1990) presented a continuum damage model, where internal damage variables are characterized by vectorial/tensorial quantities. To determine the mechanical response in the presence of damage, stiffness-damage relationships are derived from a theory with internal variables based on thermodynamical principles, wherein the damage vectors/tensors have been taken as the internal state variables.

Talreja considers two damage modes: intralaminar damage (matrix cracking) and interlaminar damage (delamination). The overall stiffness properties are reduced by the damage in the individual modes, but it is assumed that all damage modes present in the laminate, each given by its associated damage vector, do not mutually interact. Neglecting these interactions, one damage mode at a time can be accounted for and their effects can be superimposed afterwards.

Each damage mode is characterized by a damage tensor D_{ij}, which is defined as:

$$D_{ij} = \frac{1}{V}\sum_{k=1}^{n}\left[\int_S a\,n_i n_j\,dS\right]_k \tag{59}$$

where V is the representative volume element, S is the crack surface area, a is the normal component of the vector of surface activity, n_i and n_j are the crack surface normals, and n is the number of entities of the given damage mode (e.g. transverse cracks in a cross-ply laminate).

The following relations then exist:

$$\phi = \phi\left[\varepsilon_{ij}, D_{ij}^{(\alpha)}\right]$$

$$\sigma_{ij} = \rho\frac{\partial\phi}{\partial\varepsilon_{ij}} \tag{60}$$

$$\dot{D}_{ij}^{(\beta)} = \dot{D}_{ij}^{(\beta)}\left[\varepsilon_{kl}, D_{mn}^{(\alpha)}\right]$$

where ϕ is the specific Helmholtz energy, σ_{ij} is the stress tensor, ε_{ij} is the linearized strain tensor, ρ is the mass density and the Greek index α, β stands for the damage mode.
Variation of elastic moduli and Poisson's ratio were predicted for glass/epoxy and graphite/epoxy laminates (Talreja *et al* (1992)).

Bonora *et al* (1993) have presented a semi-empirical model for predicting the mechanical properties degradation of a composite laminate due to transverse matrix cracks. The constitutive equations are based on the damage model of Talreja (1986, 1990). The damage variable D is a product of three parameters, related to the crack density, length and width.

Allen *et al* (1987a, 1987b, 1990) used both phenomenological and micromechanics solutions to produce the following ply level constitutive equations, which are constructed utilizing constraints imposed by thermodynamics with internal state variables:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl} + I_{ijkl}^{\eta}\alpha_{kl}^{\eta} \tag{61}$$

where α_{kl}^{η} is an internal state variable for the damage mode η in the lamina (very similar to the definition used by Talreja) and I_{ijkl}^{η} is the damage tensor. Each damage mode is represented by the volume averaged dyadic product of the crack face displacement u_i and the crack face normal n_j:

$$\alpha_{ij}^{\eta} = \frac{1}{V_L}\int_{s_c^{\eta}} u_i^c n_j^c ds \tag{62}$$

where α_{kl}^{η} is a second order tensor internal state variable, s_c^{η} is the crack surface area, and V_L is the local representative volume, i.e., all stresses, strains and internal state variables are averaged over a local volume element.
In the case of matrix cracking, the damage evolution law due to cyclic loading in each ply is given by (Allen *et al* (1990)):

$$d\alpha_{ij}^M = \frac{d\alpha_{ij}^M}{dS} k_1 G^n dN \tag{63}$$

where k_1 and n are material properties, S is the area of matrix cracks and G is the damage dependent energy release rate within the ply. They proposed a similar evolution law for delamination in cross-ply laminates.
Lee *et al* (1989) have applied this theory for predicting stiffness reductions in glass/epoxy and graphite/epoxy laminates with matrix cracks, while Coats and Harris (1995) have used the internal states variables theory to study tension-tension fatigue of unnotched and notched graphite/bismaleimide laminates.

Ogin *et al* (1985) showed that the stiffness reduction for a $(0/90)_s$ glass fibre-reinforced laminate can be expressed over most of the range by the very simple relation:

$$E = E_0(1 - cD) \tag{64}$$

where $D = 1/2s$ is the average crack density (2s is the average crack spacing) and c is a constant. Further it is postulated that the crack growth rate is a power function of the stored elastic energy between two neighbouring

cracks in the transverse ply. By using Equation (64) the stiffness degradation rate due to transverse matrix cracking is then obtained as:

$$-\frac{1}{E_0}\frac{dE}{dN} = A\left[\frac{\sigma_{max}^2}{E_0^2(1-E/E_0)}\right]^n \tag{65}$$

where σ_{max} is the fatigue stress level, A and n are constants.

Beaumont (1987, 1990) further used the same relation (65) and made a prediction of the S-N curves by using the strain failure criterion which yields a critical value D_f of the damage variable D.

In case of delamination being the predominant damage mechanism, Beaumont has defined another damage variable D:

$$D = \frac{A}{A_0} = 2.857\left(1-\frac{E}{E_0}\right) \tag{66}$$

where A is the actual delaminated area and A_0 is the total available interfacial area between plies. The damage rate, applied to a quasi-isotropic carbon fibre laminate, was determined to be:

$$\left(\frac{dD}{dN}\right)_{R=0.1} = 9.2\cdot10^5\left(\frac{\Delta\sigma}{\sigma_{TS}}\right)^{6.4} \tag{67}$$

where R is the stress ratio, $\Delta\sigma$ is the applied stress range and σ_{TS} is the tensile strength. Again the strain failure criterion is used to determine the failure value D_f.

Carswell (1988) introduced a model for laminates with unidirectional plies. The damage variable D is related to the length of the matrix cracks in the laminate and the following damage growth rate is proposed:

$$\frac{dD}{dN} = p\,\sigma_c\,\frac{D^2}{N} \tag{68}$$

where p is a parameter, σ_c is the cyclic stress amplitude and D is related to the stiffness by the relation (64) determined previously by *Ogin et al* (1985).

Beaumont and Spearing (1990), Spearing and Beaumont (1992a, 1992b) and Spearing *et al* (1992a, 1992b) extensively studied the tensile fatigue behaviour of notched carbon/epoxy cross-ply laminates. The dominant damage modes observed were splitting in the 0° plies, delamination zones at 90°/0° interfaces the size of which is related to the split length, and transverse ply cracking in the 90° plies. The idealized damage pattern can be characterized by the split length *l* and the delamination angle α.

Based on the fatigue crack growth law for isotropic materials, the split growth rate is defined as:

$$\frac{dl}{dN} = \lambda_3\left[\frac{\Delta G}{G_c}\right]^{m/2} \tag{69}$$

where λ_3 and m are constants, ΔG is the driving force and G_c is the damage growth resistance. For an increment of split growth δl, the energy absorbed in forming new crack surfaces, is δE_{ab}:

$$\delta E_{ab} = G_s t\,\delta l + G_d(l\tan\alpha)\delta l \tag{70}$$

where G_s is the absorbed energy per unit area of split, G_d is the absorbed energy per unit area of delamination, t is the thickness of the 0° ply and α is the delamination angle at the split tip.

Some of the global energy (strain energy + potential energy) is dissipated when the split extends with a corresponding increase in specimen compliance δC:

$$\delta E_r = \frac{1}{2} P^2 \delta C \tag{71}$$

where P is the applied load. In the limit case $E_r = E_{ab}$, so that:

$$\frac{P^2}{2t} \frac{\partial C}{\partial l} = G_s + G_d \frac{l \tan \alpha}{t} \tag{72}$$

which is equivalent with $G = G_c$ (where G is the strain energy release rate and G_c is the effective toughness of the laminate). In order to evaluate the expression, $\partial C / \partial l$ must be known. This term is calculated by finite elements, because it is difficult to determine it analytically.

Then all relations are established to determine the split growth rate dl/dN. Through the use of a uniaxial tensile stress failure criterion, applied to the 0° plies, the post-fatigue laminate strength is derived (Spearing and Beaumont (1992b)). The stiffness reduction is modelled by assuming experimentally determined relations between crack spacing in the transverse plies and the longitudinal and shear moduli (Spearing *et al* (1992b)).

Caron and Ehrlacher (1997) proposed a model for fatigue microcracking in cross-ply laminates. The assumption is that the 90° plies can be discretized in sections, which are preferential sites of cracking. The strength of these sections has a random distribution. Further it is assumed that the crack propagates with a Paris law and that the residual strength degrades according to the law:

$$\frac{d \, Re}{dN} = -C \, Re^{3-\eta} \, \Delta S^\eta \tag{73}$$

where Re is the residual strength, C and η are constants, and ΔS is the stress range. From this equation, the residual lifetime can be estimated. Then an iterative procedure is started, where the stresses in the sections are calculated and compared with the residual strength. If the section breaks, stresses are redistributed and the residual life of each section is evaluated.

Experimental validation of the microcrack density has been done for tension-tension fatigue tests of $[0_2, 90_2]_s$ carbon/epoxy specimens, but no experiments have been done to check the validity of the proposed residual strength law.

Ladevèze (1990, 1992) has developed a damage mechanics model at the meso-scale, which first has been applied to static loadings. Afterwards it has been extended to fatigue loadings as well.

Laminated composites may be described at the meso-level by homogeneous layers in the thickness, and interfaces, i.e., a mechanical surface connecting two adjacent layers which depends on the relative direction of their fibres. The strain energy for the damaged material at the meso-level can then be written, in the case of plane stress assumption, as:

$$E_D = \frac{1}{2} \left[\frac{\langle \sigma_{11} \rangle_+^2}{E_1^0} + \frac{\varphi \left(\langle -\sigma_{11} \rangle_+ \right)}{E_1^0} - \left(\frac{v_{12}^0}{E_1^0} + \frac{v_{21}^0}{E_2^0} \right) \sigma_{11} \sigma_{22} \right.$$
$$\left. + \frac{\langle \sigma_{22} \rangle_+^2}{E_2^0} + \frac{\langle -\sigma_{22} \rangle_+}{(1-d')E_2^0} + \frac{\sigma_{12}^2}{(1-d)G_{12}^0} \right] \tag{74}$$

where 1, 2 and 3 are the fibre, transverse and normal direction, d and d' are two scalar internal variables which are constant within the thickness, $< . >_+$ denotes the positive part, and φ is a material function defined such that $\frac{\partial^2 E_D}{\partial^2 \sigma_{11}} = \frac{1}{E_1^c}$, where E_1^c is the compression modulus in the fibre direction.

The forces associated with the mechanical dissipation, are expressed in terms of the free energy ψ:

$$Y_d = -\rho \left.\frac{\partial \langle\langle \psi \rangle\rangle}{\partial d}\right|_{\widetilde{\sigma}=cte} = \frac{1}{2} \frac{\langle\langle \sigma_{12}^2 \rangle\rangle}{G_{12}^0 (1-d)^2}$$

$$Y_{d'} = -\rho \left.\frac{\partial \langle\langle \psi \rangle\rangle}{\partial d'}\right|_{\widetilde{\sigma}=cte} = \frac{1}{2} \frac{\langle\langle \langle \sigma_{22} \rangle_+^2 \rangle\rangle}{E_{22}^0 (1-d')^2}$$

(75)

where ρ is the mass density, $<< . >>$ denotes the mean value within the thickness, and $\widetilde{\sigma}$ denotes the chosen effective stress. The effective stress defines the coupling between classical stress and the damage state which is involved in inelastic strains. For laminates, they can be written as:

$$\widetilde{\sigma}_{12} = \frac{\sigma_{12}}{(1-d')}; \quad \widetilde{\sigma}_{22} = \frac{\sigma_{22}}{(1-d)}; \quad \widetilde{\sigma}_{11} = \sigma_{11}$$

(76)

Finally damage evolution laws are established in terms of Y_d and $Y_{d'}$.

A similar philosophy is applied for the interface modelling, where the deterioration of the mechanical surface, which ensures stress and displacement transfers from one ply to another, is described by three damage variables d, d_1 and d_2. More information on the interface modelling can be found in the work of Allix and Ladevèze (1992), Allix *et al* (1998) and Allix and Corigliano (1999).

The damage mechanics theory is applied to the static behaviour of three-dimensional composites (3D carbon/carbon) and laminate composites (carbon/epoxy).

Sedrakian *et al* (1997, 2000) have developed a fatigue damage model which is based on the Ladevèze model. Three damage variables d_{11}, d_{22} and d_{12} are respectively associated with fibre fracture, transverse matrix cracking and fibre/matrix interface failure. The strain energy and the thermodynamic variables Y_{11}, Y_{22} and Y_{12} are determined, where the free energy is now defined as:

$$\varphi = \frac{\alpha_{11}}{1+\beta_{11}} Y_{11}^{\beta_{11}} + \frac{\alpha_{22}}{1+\beta_{22}} Y_{22}^{\beta_{22}} + \frac{\alpha_{12}}{1+\beta_{12}} Y_{12}^{\beta_{12}}$$

(77)

The parameters α_i and β_i are functions of the stress ratio R, the frequency f and the stress level. The damage evolution laws are then expressed by:

$$\frac{d(d_{ij})}{dN} = \frac{\partial \varphi}{\partial Y_{ij}}$$

(78)

The theory has been applied to three-point bending tests. The experimental setup has been modelled with finite elements and two cases were distinguished: small length-to-height ratio (shear deformation) and large length-to-height ratio (flexural behaviour). For each case the stiffness loss and the maximum load for 5 % stiffness loss have been calculated.

Thionnet and Renard (1994, 1997, 1999) have developed a similar theory as the one proposed by Ladevèze in order to predict transverse cracking under fatigue. Transverse cracking was modelled at the meso-scale level and was described by a single scalar variable $\alpha_f = e/L$, where e is the thickness of the cracked ply and L is the crack spacing. Again, the thermodynamic potential ψ is constructed, as well as the thermodynamic variables. The matrix cracking is driven by a initiation criterion, which is a function of the thermodynamic variables.

The crack density was predicted for a two-level fatigue test of carbon/epoxy laminates.

Liu and Lessard (1994) used a global damage variable D, which equals $C_m D_m$ or $C_d D_d$, depending on the dominating damage type: matrix cracks or delaminations. D_m is a function of crack density and D_d is a function of delamination area, while C_m and C_d are constants that depend on material properties and laminate lay-up.

The growth rate of the global damage variable D is:

$$\frac{dD}{dN} = \frac{A.(\sigma_{max})^C}{B.D^{B-1}} \tag{79}$$

where $D = 1 - \dfrac{E}{E_0}$; A, B and C are unknown constants and σ_{max} is the maximum fatigue stress.

Four choices for predicting tension-tension allowable fatigue life and for assessing fail-safety of composite structures are proposed: a matrix-cracking criterion, a delamination-size criterion, a residual-modulus criterion and a residual-strength criterion.

For the matrix-cracking and delamination-size criterion, a maximum allowable crack density or delamination size is specified. In the case of the residual-modulus and residual-strength criterion, the additional assumption is made that the S-N relation for general laminates containing 0° plies has a power-law form:

$$K.N_f.\sigma_{max}^b = 1 \tag{80}$$

where K and b are constants, and N_f is the fatigue life at the tension-tension stress-level σ_{max}.

Shokrieh (1996) and Shokrieh and Lessard (1997a, 1997b, 1998, 2000a, 2000b) proposed a so-called 'generalized residual material property degradation model' for unidirectionally reinforced laminates. In this model three approaches are combined: (i) polynomial fatigue failure criteria are determined for each damage mode, (ii) a master curve for residual strength/stiffness is established, and (iii) the influence of arbitrary stress ratio is taken into account by use of the normalized constant-life diagram developed by Harris (1985).

First the Hashin-type fatigue failure criteria are determined for seven damage modes: fibre tension, fibre compression, fibre-matrix shearing, matrix tension, matrix compression, normal tension and normal compression. For example, for the fibre tension fatigue failure mode of a unidirectional ply under a multiaxial state of fatigue stress, the following criterion is used:

$$\left(\frac{\sigma_{xx}}{X_t(n,\sigma,\kappa)}\right)^2 + \left(\frac{\dfrac{\sigma_{xy}^2}{2E_{xy}(n,\sigma,\kappa)} + \dfrac{3}{4}\delta\sigma_{xy}^4}{\dfrac{S_{xy}^2(n,\sigma,\kappa)}{2E_{xy}(n,\sigma,\kappa)} + \dfrac{3}{4}\delta S_{xy}^4(n,\sigma,\kappa)}\right) + \left(\frac{\dfrac{\sigma_{xz}^2}{2E_{xz}(n,\sigma,\kappa)} + \dfrac{3}{4}\delta\sigma_{xz}^4}{\dfrac{S_{xz}^2(n,\sigma,\kappa)}{2E_{xz}(n,\sigma,\kappa)} + \dfrac{3}{4}\delta S_{xz}^4(n,\sigma,\kappa)}\right) = g_{F^+}^2 \tag{81}$$

where n is the number of cycles, κ is the stress ratio, δ is a parameter of material nonlinearity, $X_t(n,\sigma,\kappa)$ is the longitudinal tensile residual fatigue strength of a unidirectional ply under uniaxial fatigue loading, $S_{xy}(n,\sigma,\kappa)$ is the in-plane shear residual fatigue strength of a unidirectional ply under uniaxial shear fatigue loadings, $E_{xy}(n,\sigma,\kappa)$ is the in-plane shear residual fatigue stiffness of a unidirectional ply under uniaxial shear fatigue loading, $S_{xz}(n,\sigma,\kappa)$ is the out-of-plane shear residual fatigue strength under uniaxial shear fatigue loading and $E_{xz}(n,\sigma,\kappa)$ is the out-of-plane shear residual fatigue stiffness of a unidirectional ply under uniaxial shear fatigue loading conditions. Fatigue failure occurs when $g_{F^+} > 1$.

For all damage modes, once failure has occurred, the corresponding material properties are set to zero. These are the 'sudden material property degradation rules'.

Further they used the normalization technique and some algebraic operations to reduce the residual strength models of several researchers to one single master curve, which can be expressed as:

$$R(n,\sigma,\kappa) = \left[1 - \left(\frac{\log(n) - \log(0.25)}{\log(N_f) - \log(0.25)}\right)^\beta\right]^{\frac{1}{\alpha}} (R_s - \sigma) + \sigma \tag{82}$$

where R is the residual strength after n cycles, N_f is the number of cycles to failure, R_s is the static strength, κ is the stress ratio, σ is the applied stress and α and β are stress independent parameters. A similar master curve was derived for the residual stiffness.

Finally the fatigue life of a unidirectional ply under arbitrary state of stress and stress ratio is calculated using the normalized constant-life model developed by Harris (1985):

$$a = f \cdot \left[(1-q)(c+q) \right]^{A + B \cdot \log(N_f)} \tag{83}$$

where A, B and f are curve fitting constants, N_f is the number of cycles to failure, $a = \dfrac{\sigma_{alt}}{\sigma_t}$ is the normalized alternating stress component, $q = \dfrac{\sigma_m}{\sigma_t}$ is the normalized mean stress component and $c = \dfrac{\sigma_c}{\sigma_t}$ is the normalized compression strength.

By combining these two models with the fatigue failure criteria, a 'generalized residual material property degradation model' can be established, since both the number of cycles to failure for each stress state and the residual material properties of the unidirectional ply can be calculated. The model has been applied to the progressive fatigue damage modelling of pin/bolt-loaded unidirectional graphite/epoxy laminates.

Diao *et al* (1999) have recently modified this deterministic model to a statistical model for multiaxial fatigue behaviour of unidirectional plies.

4   CONCLUSIONS

Extensive research on fatigue modelling of fibre-reinforced composite materials has been done during the last decades. A lot of models have been proposed to predict damage accumulation and fatigue life for composites with various stacking sequences and fibre- and matrix-types under loading conditions that vary from constant-amplitude loading to spectrum loading. Nevertheless research in this domain should be addressed further attention, in order to meet the challenge of developing models with a more generalized applicability in terms of loading conditions and the materials used.

The main drawback of the fatigue life models is their dependency on large amounts of experimental input for each material, layup and loading condition (Schaff and Davidson (1997a)). Moreover these models are difficult to extend towards more general loading conditions, where multiaxial stress conditions are imposed. On the other hand most of these models are straightforward to use and do not need detailed information about actual damage mechanisms.

While the residual strength is a meaningful measure of fatigue damage, it does not allow for nondestructive evaluation as such. It is obvious to say that it is impossible to determine residual strength without destroying the specimen, which makes it very difficult to compare damage states between two specimens. Of course residual strength can be correlated with measurable manifestations of damage, but then new relations must be established between evolution of residual strength and the damage manifestation. When full-scale structural components are subjected to in-service fatigue loadings, stiffness can be a more adequate parameter as it can be measured nondestructively and the residual stiffness exhibits much less statistical scatter than residual strength (Highsmith and Reifsnider (1982), Hashin (1985), Yang *et al* (1990, 1992), Kedward and Beaumont (1992), Whitworth (1998, 2000)).

On the other hand, residual strength models possess a very natural failure criterion: if the residual strength falls to about the same level as the externally applied load, the material will fail (Harris (1985)). Residual stiffness models are dealing with different definitions of 'failure' and already in the early 70s, Salkind (1972) suggested to draw a family of S-N curves, being contours of a specified percentage of stiffness loss, to present fatigue data. Quoting from Talreja (2000) at the Second International Conference on Fatigue of Composites (June 2000), "*A reliable and cost-effective fatigue life prediction methodology for composite structures requires a physically based modelling of fatigue damage evolution. An undesirable alternative is an empirical approach. A major obstacle to developing mechanistic models for composites is the complexity of the fatigue damage mechanisms, both in their geometry and the details of the evolution process. Overcoming this obstacle requires insightful simplification that allows the use of well-developed mechanics modelling tools without compromising the essential physical nature of the fatigue process*".

In the authors' belief as well, the progressive damage models are the most promising tool because they quantitatively account for the progression of damage in the composite structure. Of course damage growth must be correlated with residual mechanical properties, as was demonstrated by Shokrieh and Lessard (1997a, 1997b, 2000a, 2000b) who proved that a more generalized approach is possible, integrating residual strength and stiffness theory, failure criteria for different damage mechanisms and constant-life analysis.

As regards to implementation in numerical software, the damage accumulation models which correlate damage with the degradation of material properties, could be used for real structures as well. Indeed, as mentioned before, the gradual deterioration of a fibre-reinforced composite – with a loss of stiffness in the damaged zones – leads to a continuous redistribution of stress and a reduction of stress concentrations inside a structural component (Allen *et al* (1990), Shokrieh and Lessard (2000a), Van Paepegem and Degrieck (2000a, 2001)). As a consequence an appraisal of the actual state or a prediction of the final state of the composite structure requires the simulation of the complete path of successive damage states.

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BIOGRAPHICAL SKETCHES

Joris Degrieck has a MS degree in mechanical engineering (1982, Ghent University) and a PhD degree in applied science (mechanical engineering) (1990, Ghent University) which was awarded the Vreedenburgh-Prize (TU-Delft, Netherlands, 8 april 1991). He is a full professor at the Department of Mechanical Construction and Production of Ghent University, teaching courses on mechanics of fibre-reinforced materials and kinematics and dynamics of machines. His research interests are in the mechanics of composite materials, the development of damage models for composites under dynamic loading (both fatigue and impact, both numerically and experimentally), and experimental characterization of composite materials by means of optical fibre sensors and ultrasonic polar scans. He is a member of SAMPE and BSMEE.

Wim Van Paepegem is research assistant at the Department of Mechanical Construction and Production of Ghent University. He has a MS degree in civil engineering (Ghent University, 1998). His research interests are in the areas of fatigue damage modelling of fibre-reinforced composite materials and finite element modelling of the fatigue behaviour of these materials.