Van Paepegem, W. and Degrieck, J. (2001). Modelling Strategies for Fatigue Damage Behaviour of Fibre-reinforced Composites. *European Journal of Mechanical and Environmental Engineering*, 46(4), 217-227.

Modelling Strategies for Fatigue Damage Behaviour of Fibre-reinforced Polymer Composites

Wim Van Paepegem, Joris Degrieck
Ghent University, Dept. of Mechanical Construction and Production
Sint-Pietersnieuwstraat 41, 9000 Gent, Belgium
E-mail: Wim.VanPaepegem@rug.ac.be

Abstract

Fibre-reinforced composite materials are finding increasingly application in aerospace industry, naval and automotive applications and other high-tech designs, because of their high specific stiffness and strength. Although the fatigue behaviour of fibre-reinforced composite materials has been studied for a long time, it is still not possible to make adequate predictions about the fatigue life and the degradation of stiffness and strength without extensive experimental investigation. Hence large safety factors are adopted when designing with composite materials. As a consequence reliable numerical simulations would result in large savings in budget and time. Moreover the implementation of the fatigue models in a finite element code is indispensable in order to apply them to the simulation of full-scale structural composite components.
This paper presents an overview of the modelling strategies for fatigue damage of fibre-reinforced polymer composite materials. It draws attention to pitfalls concerning adequate calculation of laminar stresses and modelling of boundary conditions, and finite element implementation of fatigue damage models and inherent numerical difficulties.

Keywords: Fatigue; Composites; Finite Element Analysis; Residual Stiffness.

1. Introduction

Composite materials are a combination of two or more chemically different phases with a distinct interface between them. Thus a composite is heterogeneous and consists of two or more materials which together produce desirable properties that cannot be achieved with any of the constituents alone. Fibre-reinforced composite materials, for example, consist of high strength and high modulus *fibres* in a *matrix* material. In these composites, fibres are the principal load-carrying components, and the matrix material keeps the fibres together, acts as a load-transfer medium between fibres, and protects fibres from being exposed to the environment (e.g. moisture, toxic agents) [1-3].
Fibre-reinforced composite materials for structural applications are often made in the form of a thin layer, called *lamina*. A lamina is a macro unit of material whose material properties are determined through appropriate laboratory tests. Structural elements, such as bars, beams or plates are then formed by stacking the layers to achieve desired overall strength and stiffness. These structural elements are called *laminates*. Fibre orientation in each lamina and stacking sequence of the layers in the laminate can be chosen to achieve desired strength and stiffness for a specific application.
A wide variety of fibres and matrix materials are now available for use in advanced composites. The selection of the specific fibre and matrix to be used in a composite is not arbitrary. The two (or more) phases of a composite must be carefully chosen if the composite material is to be structurally efficient. The composite generally must be resistant to debonding at the fibre/matrix interface, and it must also be resistant to fibre breakage and matrix cracking. However, in applications where it is desired to dissipate energy during the failure process (such as in crashworthy or impact-resistant structures), progressive fibre failure and fibre/matrix debonding (damage development) are positive features because they dissipate energy. Thus, a major challenge for the mechanics and materials community is to understand the factors influencing damage development and to know how to design for it under severe environmental and mechanical loading conditions, including the fabrication phase as well as

the in-service phase.

Long fibre-reinforced polymer composites have a rather good rating as regards to life time in fatigue. The same does not apply to the number of cycles to initial damage nor to the evolution of damage. These composite materials are inhomogeneous and anisotropic, and their behaviour under fatigue is more complicated than that of homogeneous and isotropic materials such as metals. The main reasons for this are the different types of damage that can occur (e.g. fibre fracture, matrix cracking, matrix crazing, fibre buckling, fibre-matrix interface failure, delaminations), their interactions and their different growth rates.

Among the parameters that influence the fatigue performance of composites are:

- fibre type (glass, carbon, aramid,…),
- matrix type (thermosetting or thermoplastic resin),
- type of reinforcement structure (unidirectional, mat, fabric, braiding,...),
- laminate stacking sequence,
- environmental conditions (mainly temperature and moisture),
- loading conditions (stress ratio R, cycling frequency,...) and boundary conditions.

As a consequence the microstructural mechanisms of damage accumulation, of which there are several, occur sometimes independently and sometimes interactively, and the predominance of one or other of them may be strongly affected by both material variables and testing conditions.

There are a number of differences between the fatigue behaviour of metals and fibre-reinforced polymer composites. In metals the stage of gradual and invisible deterioration spans nearly the complete life time. No significant reduction of stiffness is observed during the fatigue process. The final stage of the process starts with the formation of small cracks, which are the only form of macroscopically observable damage. Gradual growth and coalescence of these cracks quickly produce a large crack and final failure of the structural component. As the stiffness of a metal remains quasi unaffected, the linear relation between stress and strain remains valid, and the fatigue process can be simulated in most common cases by a linear elastic analysis and linear fracture mechanics.

In a fibre-reinforced polymer composite, damage starts very early and the extent of the damage zones grows steadily, while the damage type in these zones can change (e.g. small matrix cracks leading to large size delaminations). The gradual deterioration of a fibre-reinforced composite – with a loss of stiffness in the damaged zones – leads to a continuous redistribution of stress and a reduction of stress concentrations inside a structural component. As a consequence an appraisal of the actual state or a prediction of the final state (when and where final failure is to be expected) requires the simulation of the complete path of successive damage states.

In the following paragraphs, the present research of the authors will be used as a guideline to discuss several problems with modelling fatigue behaviour of fibre-reinforced composite materials.

2. Experimental procedures

2.1. Material

The material used in this research programme was a glass fabric/epoxy composite. The fabric was a Roviglass R420 plain woven glass fabric (Syncoglas) and the epoxy was Araldite LY 556 (Ciba-Geigy). The plain woven glass fabric was stacked in eight layers and two different stacking sequences were used: the first type is denoted as $[\#0^o]_8$, where ‘0°’ means that the warp direction of each of the eight layers has been aligned with the loading direction and where the symbol ‘#’ refers to the fabric reinforcement type, while the angle between the warp direction of all layers of the second type and the loading direction is 45º (denoted as $[\#45^o]_8$). All composite specimens were manufactured using the resin-transfer-moulding technique. After curing they had a thickness of 2,72 mm. The samples were cut to dimensions of 145 mm long by 30 mm wide on a water-cooled diamond saw.

The in-plane elastic properties of the $[\#0°]_8$ composite laminates were determined using the dynamic modulus identification method described by Sol et al. [4,5]. They are listed in Table 1.

Table 1 Measured in-plane elastic properties of the $[\#0°]_8$ composite laminates

E_{11} [GPa]	24,57
E_{22} [GPa]	23,94
ν_{12} [-]	0,153
G_{12} [GPa]	4,83

2.2. Testing procedure

Although fatigue experiments in pure tension and compression are most often used in fatigue research [6-8], bending fatigue experiments were preferred because they allow to test the finite element implementation in more complicated and hence more realistic conditions [9-11].

A schematic drawing of the setup is shown in Figure 1.

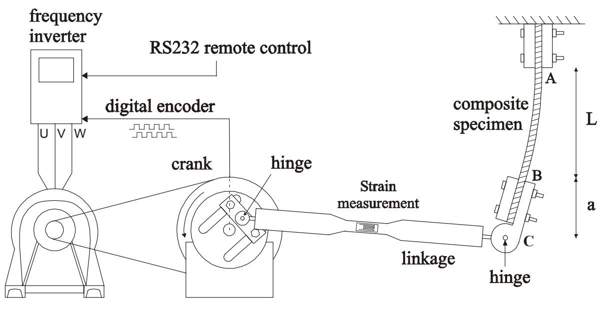

Figure 1 Schematic drawing of the experimental setup for bending fatigue.

The outcoming shaft of the motor has a rotational speed of 185 rpm. The power is transmitted by a V-belt to a second shaft, which provides a fatigue testing frequency of 2,2 Hz. The influence of frequency can be assumed small in this range of values [12]. The power transmission through the V-belt ensures that the earthing of the motor and the earthing of the measurement system are electrically isolated, which is an advantage mainly with metallic or carbon fibre-reinforced specimens.

The second shaft bears a crank-linkage mechanism, which imposes an alternating displacement on the hinge (point C) that connects the linkage with the lower clamp of the composite specimen. At the upper end the specimen is clamped. Hence the sample is loaded as a composite cantilever beam. The amplitude of the imposed displacement is a controllable parameter and the adjustable crank allows to choose between single-sided and fully-reversed bending, i.e. the deflection can vary from zero to the maximum deflection in one direction, or in two opposite directions, respectively.

Due to the varying bending moment along the specimen length, the stresses and strains are different in each material point and they are redistributing continuously during fatigue life. Therefore the out-of-plane displacement profile is a very important indicator of the damage state. Indeed, it is observed that for the $[\#0°]_8$ specimens, the damage at the clamped end can be such that a complete hinge is formed and that the strains and stresses in the rest of the specimen are almost zero. To record this displacement profile in the maximum deformed state, it was necessary to develop a mechanism to hold the specimen fixed in this state, because recording the profile while the test keeps running at a frequency of 2,2 Hz, gives rise to some practical problems. A rotary digital encoder was attached to the second shaft. Its angular position (relative to a certain reference angle) is directly related with the loading path of the composite specimen. The frequency inverter reads the signal from the rotary encoder and can stop and hold the motor at a predetermined angular position of the encoder. The commands for the inverter are controlled by a computer and sent through a RS232 communication line. The out-of-plane displacement profile can then be recorded with a digital camera system.

Fatigue experiments were performed with different values of the imposed displacement, as well as with single-sided and fully-reversed bending. To characterize each experiment, the 'displacement ratio' $R_d = \frac{u_{min}}{u_{max}}$ (analogous to the stress ratio R) is defined, whereby the minimum deflection is not necessarily zero. When the displacement u_{max} and the length L between the both clamps are further given too, all parameters of the fatigue experiment are known.

Figure 2 shows the force-cycle history for a $[\#0°]_8$ and $[\#45°]_8$ specimen, subjected to single-sided bending with u_{max} = 34,4 mm. The abscissa contains the number of cycles; the ordinate axis shows the force (Newton), which is measured by a strain gauge bridge (Figure 1).

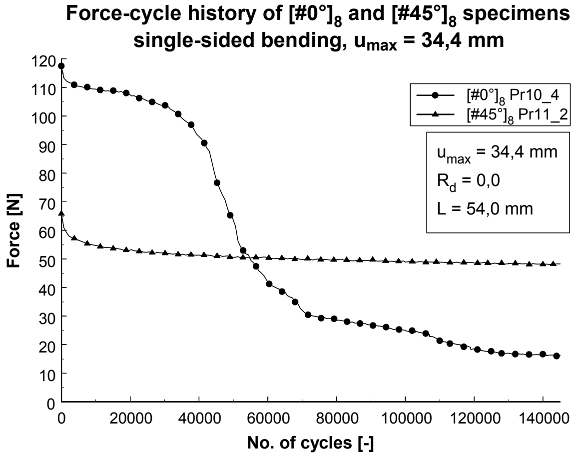

Figure 2 Force-cycle histories for $[\#0°]_8$ and $[\#45°]_8$ specimens (single-sided bending, u_{max} = 34,4 mm).

It is clearly demonstrated that the fatigue behaviour of both specimen types is very different. With regard to the $[\#0°]_8$ specimen, a hinge is formed at the clamped cross-section (which corresponds with the sharp decline in the force-cycle history in Figure 2) and the strains in the parts remote from the fixation are reduced to nearly zero. On the other hand the out-of-plane displacement profile of the $[\#45°]_8$ specimen remains nearly the same, as well as the force necessary to impose the bending displacement (Figure 2).

3. Modelling of the experimental fatigue loading conditions

Before implementing a fatigue damage model, it is very important to model the experimental loading conditions as realistically as possible. The parameter that was used for validating the finite element modelling of the in-service loading conditions, was the difference between the experimentally measured and theoretically calculated value of the bending force. To be sure that the elastic properties used in the finite element simulations were in close agreement with the real ones, the bending force has been evaluated for the $[\#0°]_8$ glass/epoxy laminates, because the in-plane elastic properties of the $[\#0°]_8$ composite laminates were well known (Table 1).

Two important aspects will be considered: (i) the correct calculation of the stresses in the individual plies, and (ii) the adequate modelling of the boundary conditions.

3.1. Laminate structural analysis theories

When the use of composite materials is restricted to secondary, noncritical structural components, the main purpose of an analysis is to determine the global response of the laminated component, for example, gross deflections, critical buckling loads, fundamental vibration frequencies and associated mode shapes. Such global behaviour can often be accurately determined using relatively simple laminate theories (e.g. the classical laminated plate theory and the first order shear deformation theory), especially for very thin laminates. However, if laminated composite materials undergo the transition from secondary structural components to primary critical structural components, the goals of analysis must be broadened to include a highly accurate assessment of localized regions where damage initiation is likely. The simple laminate theories that have proven to be adequate for modelling secondary structures, are of limited value in modelling primary structures for two reasons [13]:

- most primary structural components are considerably thicker than secondary components, thus even the determination of the global response may require a refined laminate theory that accounts for thickness effects, which are of two types: (i) multi-layer problems which require stringent fidelity in calculation of stresses, and (ii) a low span-to-thickness ratio in which case shear deformation theories are required to provide accurate prediction of deflections, buckling loads, and vibration frequencies. Indeed, due to their low transverse shear stiffness, composite laminates often exhibit significant transverse shear deformation at lower span-to-thickness ratio than do similar homogeneous isotropic plates and shells. As a consequence, the transverse shear deformation plays a much more important role in reducing the effective flexural stiffness of laminated plates made of these composites than in the corresponding metallic plates. Also, the transverse shear deformation was found to be important in predicting the delamination type of failure in multi-layered composite structures [14];
- the assessment of localized regions of potential damage initiation requires determination of the 3-D state of stress and strain at the ply level. The simple laminate theories are most often incapable of determining the 3-D stress field at the ply level. Thus the analysis of primary composite structural components may require the use of a layerwise laminate theory or of 3-D elasticity theory.

The need for more accurate computational models for multi-layered laminated plates has led to the development of a variety of 2-D shear deformation theories. Together with the conventional 3-D elasticity theory, these theories can be grouped into three general categories [14]:

- theories based on replacing the laminated plate by an equivalent single-layer anisotropic plate and introducing global displacement, strain and/or stress approximations in the thickness direction;
- discrete layer theories based on piecewise approximations in the thickness direction;
- the full 3-D elasticity theory which is of course the most general theory for assessing the stress state of the laminate.

Equivalent single layer theories

The equivalent single layer theories are those in which a heterogeneous laminated plate or shell is treated as a statically equivalent single layer, having (possibly) complex constitutive behaviour, thus reducing the 3-D continuum problem to a 2-D one. The equivalent single layer theories are developed by assuming that the displacement field is at least C^1-continuous (i.e. the function and its derivative are continuous) through the thickness of the laminate [13].

This first category includes the classical laminated plate theory (CLPT), the Reissner-Bollé-Mindlin type first-order shear deformation theories and higher-order theories.

Both the classical laminated plate theory and first order shear deformation theory yield finite element models that are economical in terms of the number of degrees of freedom used. However both finite element models have certain drawbacks [13]. For example, finite element models of the classical laminated plate theory require C^1-continuity of the transverse displacement, which complicates the development of conforming elements and inhibits their use with other commonly used finite elements. In contrast, finite elements based on the first order shear deformation theory have the advantage of requiring only C^0-continuity of all primary variables, because, unlike in the classical laminated plate theory, the shear rotations ψ_x and ψ_y of the transverse normal are independent of the transverse displacement w_0 [15]. However, they can exhibit spurious transverse shear stiffness (or locking) as the laminate becomes thin. The effect of spurious shear stiffnesses can be alleviated by using a reduced quadrature to compute certain terms in the element stiffness matrix (i.e. a selective integration scheme), or by using higher order elements [16,17].

Higher order equivalent single layer laminated plate theories use higher order polynomials (quadratic or cubic) in the expansion of the displacement components through the thickness of the laminate. The additional unknowns introduced in second-order and third-order theories are difficult to interpret in physical terms. The third-order theories provide a slight increase in accuracy relative to the first order shear deformation theory solution, at the expense of a significant increase in computational effort. Further, finite element models of third order theories that satisfy the vanishing of transverse shear stresses on the boundary planes have the disadvantage of requiring C^1-continuity of the transverse displacement component [13].

In general, despite their inherent simplicity and low computational cost, the equivalent single layer models often provide a sufficiently accurate description of the global response of thin to moderately thick laminates. Of the equivalent single layer models, finite element models based on the first order shear deformation theory appear to provide the best compromise of solution accuracy, solution economy, model simplicity, and compatibility with other finite element displacement models. However, the equivalent single layer models have several serious limitations that prevent them from being used to solve the whole spectrum of composite laminate problems. First, the accuracy of the global response predicted by the equivalent single layer models deteriorates as the laminate becomes thicker. Secondly, the equivalent single layer models are often incapable of accurately describing the state of stress and strain at the ply level near geometric and material discontinuities or near regions of intense loading – the areas where accurate stresses are most needed. Finally, the equivalent single layer models cannot model the kinematics of delamination [13].

Layerwise theories

In contrast to the equivalent single layer theories, the layerwise theories are developed by assuming that the displacement field is only C^0-continuous through the laminate thickness [13]. Thus the displacement components are continuous through the laminate thickness, but the transverse derivatives of the displacements may be discontinuous at various points through the thickness, thereby allowing for the possibility of continuous transverse stresses at interfaces separating dissimilar materials. Layerwise displacement fields provide a much more kinematically correct representation of moderate to severe cross-sectional warping, associated with the deformation of moderately thick to very thick laminates.

Layerwise theories can be divided into two categories: partial layerwise and full layerwise theories.

Partial layerwise theories are based on piece-wise linear variation of the inplane displacement components and a constant transverse displacement through the thickness (i.e. the transverse shear strain is piece-wise constant, while the transverse normal strain is zero through the thickness). Such layerwise theories are referred to as partial layerwise theories since the transverse normal displacement does not have a layerwise representation.

Despite the success of the partial layerwise laminate models, these models are not capable of accurately determining interlaminar stresses near discontinuities such as holes or cut-outs, traction free edges, and delamination fronts. In modelling these localized effects, inclusion of the transverse normal strain is important for two reasons. First of all, the transverse normal stress is usually a significant, if not dominant, stress component in these regions. Secondly, layerwise models that neglect transverse normal strain do not satisfy traction free boundary conditions for transverse shear stresses at the laminate edge.

In contrast to the partial layerwise theories, full layerwise theories use layerwise expansions for all three displacement components, and thus include both discrete layer transverse shear effects and transverse normal effects. The generalized laminate plate theory of Reddy [18] is the most valuable theory in this domain. This theory is based on the idea that the displacements can be represented as a linear combination of products of functions of the in-plane coordinates and of functions of the thickness coordinate.

The finite element models of full layerwise theories are capable of achieving the same level of solution accuracy and require the same number of degrees of freedom as a conventional 3-D finite element model. Thus, it is most often impractical to model an entire laminate with full layerwise elements. However the use of full layerwise models has two advantages over conventional 3-D finite element models.

First, the element stiffness matrices can be computed much faster for layerwise elements by performing the in-plane and out-of-plane integration separately. Indeed, the layerwise model assumes that the displacements, material properties and element geometry can be approximated by a sum of conveniently separable interpolation functions (i.e. each individual 3-D interpolation function can be written as the product of a 2-D interpolation function and a 1-D interpolation function). This restriction allows the layerwise model to use separated numerical integration to compute all element volume integrals. The results from a single integration through the thickness can then be used at each Gauss-point in the subsequent in-plane integration. This separated integration allows the element stiffness matrix to be computed using only a fraction of the operations required to form the stiffness matrix for a conventional 3-D finite element [17].

Secondly, the 2-D data structure of the full layerwise models permits a much easier coupling with conventional equivalent single layer laminate models and partial layerwise laminate models [17]. Indeed, to capture the localized 3-D stress fields in a tractable manner, it is usually necessary to resort to a simultaneous multiple model (SMM) approach in which different subregions of the structure are described with different types of mathematical models. This approach allows to model the global response of the laminate using a conventional 2-D plate or shell model, while localized regions of intense loading can be modelled with a layerwise 2-D plate or shell model or a 3-D finite element model. While such simultaneous multiple methods are simple in concept, the actual implementation of the techniques is complicated and cumbersome, mainly due to the need for maintaining displacement continuity across subregion boundaries separating incompatible subdomains [19]. The 2-D data structure of the full layerwise models simplifies the integration of the subdomains in the laminate structure.

Three-dimensional elasticity theory

A finite element model based on the 3-D elasticity theory requires, ideally, at least one element through the thickness of each layer. To keep the element aspect ratios within reasonable limits to avoid numerical problems of 'locking', a large number of 3-D elements is required to model a laminate and the thickness of the individual lamina dictates the aspect ratio of an element. The cost of analysis precludes the sole use of

Table 2 Comparison of the different finite element models for the bending setup.

FE model type	No. of elements	Bending force [N]	CPU time
2D plane strain, complete fixation	445	155,1	0'17''
2D plane strain, clamping surfaces	517	141,2	0'46''
3D symmetry model, complete fixation	1461	139,2	32'45''
3D symmetry model, complete fixation, inertia forces	1461	138,9	4 u 35'57''
3D symmetry model, clamping surfaces, no geometrical non-linearities	1765	146,7	21'53''
3D symmetry model, clamping surfaces	1765	120,8	40'44''

3-D elements in practical problems. To reduce the computational effort involved with the 3-D elements, one can use the 'sublaminate' concept, in which several layers are modelled using one finite element through the thickness. The material properties of the sublaminate are obtained by integrating the laminae properties through the thickness of the sublaminate in the same way as is done in the single-layer continuum theories [16].
On the other hand the 3-D elasticity theory still provides the most accurate means to calculate the complex stress fields at free edges, cut-outs, bolted joints,… Since it is very important to calculate the correct stress state when developing models for composite behaviour, the 3-D elasticity theory is used in this research and each layer of the laminate is modelled by one element through the thickness.

3.2. Modelling of the boundary conditions

As was already mentioned, another aspect which must be considered carefully, is the modelling of the fixation. In most cases, the fixation is modelled in finite elements by fixing the displacement and rotational degrees of freedom of the nodes in the clamped cross-section. This way of modelling will be referred to as "complete fixation". However in real testing conditions (Figure 1), the specimen is clamped between two plates and a prestressing force is applied by nuts and bolts. This allows for some rotation of the specimen cross-section even inside the fixation.
Table 2 illustrates the influence of several assumptions on the outcome of the structural analysis (the calculations were done on a 300 MHz Sun UltraSparc Workstation with 256 MB RAM). The imposed harmonic displacement was chosen rather large in order to assess the effect of geometrical non-linearities; the amplitude u_{max} was 34,4 mm. The corresponding maximum bending force, measured by a strain gauge bridge, was 117,8 Newton at the first loading cycle (see Figure 2). 2-D and 3-D meshes have been used with "complete fixation" (fixing all nodes in the clamped cross-section) and "clamping surfaces" (modelling of the clamping plates with prestressing force). All simulations are quasi-static analyses, except the fourth simulation, which takes into account the inertia forces during fatigue loading.

From the third and fourth simulation it is confirmed that a quasi-static analysis is sufficient to evaluate the stresses. Indeed, since the fatigue experiments are performed at a frequency of 2,2 Hz and the mass of the actuating parts is very small due to the limited forces in bending, the inertia forces are negligible. The third and sixth simulation clearly illustrate the importance of how the clamping is exactly modelled. The sixth simulation is the more realistic and is indeed in close agreement with the experimentally measured force of 117,8 N. The small difference remaining can be due to small deviations in the value of the elastic properties, as the laminates are produced by resin-transfer-moulding. Figure 3 shows the finite element mesh for the 3-D analysis with full modelling of the clamped surfaces. Due to the symmetry conditions with respect to the (x,z)-plane, only one half of the specimen has to be modelled. This finite element mesh was then used for all subsequent work.

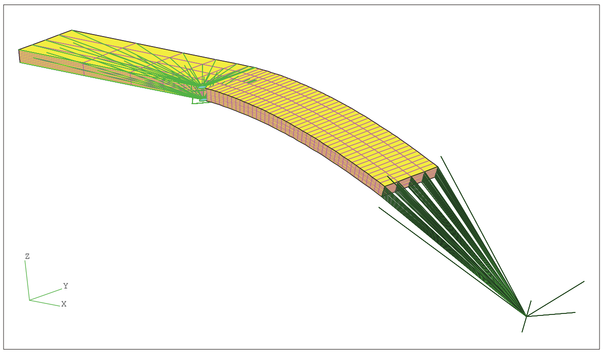

Figure 3 Finite element model of the bending fatigue setup.

4. Fatigue modelling strategies

A rigorous classification of the most important fatigue models and life time prediction methodologies for fatigue testing of fibre-reinforced polymers is difficult, but a workable classification can be based on the classification of fatigue criteria by Sendeckyj [20]. According to Sendeckyj, fatigue criteria can be classified in four major categories: the macroscopic strength fatigue criteria, criteria based on residual strength and those based on residual stiffness, and finally the criteria based on the actual damage mechanisms.

A similar classification can be used to classify the large number of existing fatigue models for composite laminates and consists of three major categories: fatigue life models, which do not take into account the actual degradation mechanisms but use S-N curves or Goodman-type diagrams and introduce some sort of fatigue failure criterion; phenomenological models for residual stiffness/strength; and finally progressive damage models which use one or more damage variables related to measurable manifestations of damage (transverse matrix cracks, delamination size).

Although the fatigue behaviour of long fibre-reinforced polymer composites is fundamentally different from the behaviour exposed by metals, many models have been established which are based on the well-known S-N curves. These models make up the first class of so-called ‘fatigue life models’. This approach requires extensive experimental work and does not take into account the actual damage mechanisms, such as matrix cracks and fibre fracture.

The second class comprises the phenomenological models for residual stiffness and strength. These models propose an evolution law which describes the (gradual) deterioration of the stiffness or strength of the composite specimen in terms of macroscopically observable properties, as opposed to the third class of progressive damage models, where the evolution law is proposed in direct relation with specific damage. Residual stiffness models account for the degradation of the elastic properties during fatigue. Stiffness can be measured frequently during fatigue experiments, and can be measured without further degrading the material [21]. The model may be deterministic, in which a single-valued stiffness property is predicted, or statistical, in which predictions are for stiffness distributions. The other approach is based on the composite’s strength. In many applications of composite materials it is important to know the residual strength of the composite structure, and as a consequence the remaining life time during which the structure can bear the external load. Therefore the so-called ‘residual strength’ models have been developed, which describe the deterioration of the initial strength during fatigue life. From their early use, strength-based models have generally been statistical in nature. Most commonly, two-parameter Weibull functions are used to describe the residual strength and probability of failure for a set of laminates after an arbitrary number of cycles.

Since the damage mechanisms which govern the fatigue behaviour of fibre-reinforced composites, have been studied intensively during the last decades, a last class of models have been proposed which describe the deterioration of the composite material in direct relation with specific damage (e.g. transverse matrix cracks, delamination size). These models correlate one or more properly chosen damage variables to some measure of the damage extent, quantitatively accounting for the progression of the actual damage mechanisms. These models are often designated as ‘mechanistic’ models.

Summarized, fatigue models can be generally classified in three categories: the fatigue life models, the phenomenological models for residual stiffness/strength, and the progressive damage models.

One of the important outcomes of all established fatigue models is the life time prediction. Each of the three categories uses its own criterion for determining final failure and as a consequence for the fatigue life of the composite component.

As mentioned the fatigue life models use the information from S-N curves or Goodman-type diagrams, and they introduce a fatigue failure criterion which determines the fatigue life of the composite specimen. Regarding the characterization of the S-N behaviour of composite materials, Sendeckyj [22] advises to take into account three assumptions:

- the S-N behaviour can be described by a deterministic equation,
- the static strengths are uniquely related to the fatigue lives and residual strengths at runout (termination of cyclic testing). An example of such a relationship is the commonly used ‘strength-life equal rank assumption’ which states that for a given specimen its rank in static strength is equal to its rank in fatigue life [23,24],
- the static strength data can be described by a two-parameter Weibull distribution.

Residual strength models have in fact an inherent ‘natural failure criterion’: failure occurs when the applied stress equals the residual strength [25,29]. In the residual stiffness approach, fatigue failure is assumed to occur when the modulus has degraded to a critical level, which has been defined by many investigators. Hahn and Kim [26] and O’Brien and Reifsnider [27] state that fatigue failure occurs when the fatigue secant modulus degrades to the secant modulus at the moment of failure in a static test. According to Hwang and Han [28], fatigue failure occurs when the maximum fatigue strain reaches the static ultimate strain.

Damage accumulation models and life time prediction methodologies are very often inherently related, since the fatigue life can be predicted by establishing a fatigue failure criterion which is imposed to the damage accumulation model. For specific damage types, the failure value of the damage variable(s) can be determined experimentally.

5. Residual stiffness model

The main drawback of the fatigue life models is their dependency on large amounts of experimental input for each material, layup and loading condition [29]. Moreover these models are difficult to extend towards more general loading conditions, where multiaxial stress conditions are imposed. On the other hand most of these models are straightforward to use and do not need detailed information about actual damage mechanisms.

While the residual strength is a meaningful measure of fatigue damage, it does not allow for nondestructive evaluation as such. It is obvious to say that it is impossible to determine residual strength without destroying the specimen, which makes it very difficult to compare damage states between two specimens. Of course residual strength can be correlated with measurable manifestations of damage, but then new relations must be established between evolution of residual strength and the damage manifestation. When full-scale structural components are subjected to in-service fatigue loadings, stiffness can be a more adequate parameter as it can be measured nondestructively and the residual stiffness exhibits much less statistical scatter than residual strength [21,30-34].

In previous work of the authors [35,36], it has been clearly demonstrated that the gradual deterioration of a fibre-reinforced composite – with a typical loss of stiffness in the damaged zones – leads to a continuous redistribution of stresses and strains, and to a reduction of stress concentrations inside a structural component. Therefore a residual stiffness model was adopted to simulate the fatigue damage behaviour of the composite specimens. It was shown that a fatigue damage model, similar to the one proposed by Sidoroff and Subagio [37], is capable of simulating the fatigue damage behaviour of the composite specimens:

$$\frac{dD}{dN} = \begin{cases} \dfrac{A \cdot \left(\dfrac{\Delta\sigma}{\sigma_{TS}} \right)^c}{(1-D)^b} & \text{in tension} \\ 0 & \text{in compression} \end{cases} \qquad (1)$$

where:
- D : local damage variable
- N : number of cycles
- $\Delta\sigma$: amplitude of the applied loading
- σ_{TS} : tensile strength
- A, b and c : three material constants

The local damage variable D is associated with the longitudinal stiffness loss. The damage value is lying between *zero* (virgin state of the material) and *one* (complete failure of the material). The stresses and strains are related by the commonly used equation in continuum damage mechanics (with E_0 being the undamaged modulus):

$$\sigma = E_0.(1-D).\varepsilon \qquad (2)$$

The assumption that damage is not growing in the regions subjected to compressive stresses, is justified because with the experiments, no micro-buckling nor any macroscopically significant damage could be observed at the surface that was subjected to compressive stresses.

6. Finite element analysis

An indispensable need to validate a proposed fatigue damage model is its implementation into finite element software. Although the vast majority of the fatigue damage models are developed on the basis of laboratory experiments, their applicability should be validated against fatigue tests on full-scale structural components. Therefore it is mandatory that the fatigue damage model has been implemented into finite elements.

The finite element implementation of the fatigue damage model deals with two important, but contradictory demands:
- in order to correctly predict the damage and residual stiffness of the composite construction after a certain number of cycles, the simulation should trace the complete path of successive damage states to keep track of the continuous stress redistribution,
- as it is impossible to simulate each of the hundreds of thousands of loading cycles for a real construction, or even for a part of it, the finite element calculations should be fast and computationally efficient.

To meet both requirements, the authors have chosen to adopt a *cycle jump* approach [38], which means that the computation is done for a certain set of loading cycles at deliberately chosen intervals, and that the effect on the stiffness degradation of these loading cycles is extrapolated over the corresponding intervals in an appropriate manner. Figure 4 illustrates the *cycle jump* principle.

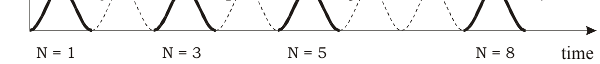

Figure 4 Illustration of the *cycle jump* principle.

To this purpose, each Gauss-point has been assigned – besides the damage variable D – a second state variable NJUMP1, which is the number of cycles that could be jumped over without losing reliability and accuracy for that particular Gauss-point.

There are several ways to determine the value of the local *cycle jump* NJUMP1. A criterion could be imposed to the stress components, to the damage variable(s) or to some weighted combination of them. The choice of the authors to impose such a criterion to the *damage* value is not randomly chosen. Indeed, although the damage curves of the Gauss-points can be rather different in shape, they have two important advantages for extrapolation, compared to the stress curves:

- the value of the damage variable D is always lying between known values; *zero* (virgin state material) and *one* (complete failure of the material),
- the gradient dD/dN must be positive or zero. The curve can never decrease, because the damage state reached cannot be reversed anymore. On the other hand, depending on stress redistributions, stresses can increase or decrease, without any foreknowledge.

These properties of the damage curves remain when considering complex stress-cycle histories of real in-service fatigue loadings. In such multi-axial loading conditions, several stress components will affect the material's fatigue behaviour, while each of them can decrease or increase at different moments in fatigue life. Extrapolating these stress-cycle histories is a hazardous job, because it is almost impossible to define a common procedure that can cope with the extrapolation of such dissimilar stress-cycle histories.

Bearing in mind these considerations, the damage value has been found appropriate to assess the local *cycle jump* NJUMP1. The criterion for the damage value is then defined by imposing a maximum increase in damage dD/dN for each particular Gauss-point when the calculation would proceed for NJUMP1 cycles. When the increase dD/dN is limited to for example 0,01, this is equivalent to a stepwise integration of the damage evolution law for that Gauss-point by dividing the ordinate axis of the damage-cycle history into 100 segments.

After looping over all the Gauss-points, a cumulative relative frequency distribution of the NJUMP1 values is calculated and the overall *cycle jump* NJUMP (which will be applied to the whole finite element mesh) is determined as a fractile of this frequency distribution. By decreasing the upper threshold for dD/dN for each Gauss-point, the damage evolution law dD/dN will be integrated more accurately, but the global NJUMP – a fractile of the cumulative frequency distribution of all NJUMP1 values – will be smaller and the calculation will proceed more slowly.

The finite element approach was implemented in the commercial finite element code SAMCEF^{TM} and the simulations were done on a Sun Workstation. Figure 5 shows the results of the experimental data and the finite element simulation for a $[\#45°]_8$ specimen, subjected to single-sided bending with u_{max} = 32,3 mm. Only 107 *cycle jumps* (i.e. finite element runs) were necessary to simulate the 400 000 loading cycles.

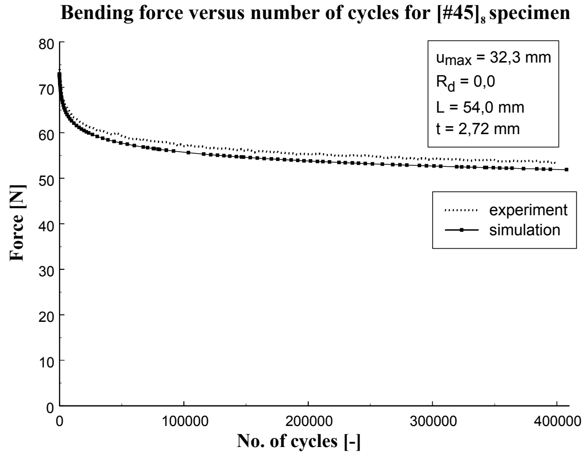

Figure 5 Experimental results versus finite element simulation for a $[\#45°]_8$ composite specimen.

When the distribution of normal stress in the clamped cross-section of the composite specimen is plotted for increasing numbers of loading cycles (Figure 6), it is seen that damage, and as a consequence stiffness loss, is affecting the stress distribution in the cross-section.

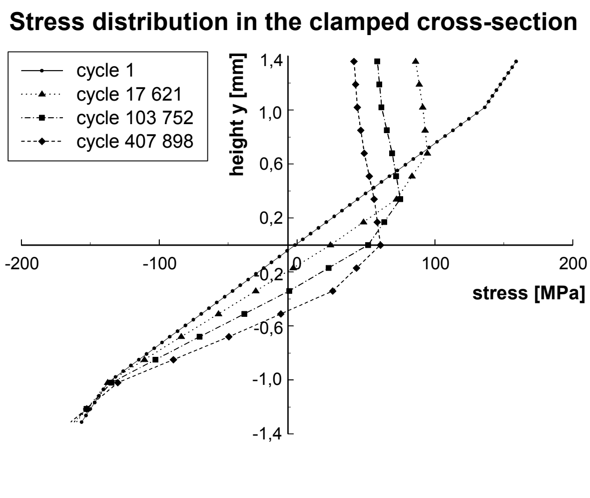

Figure 6 Distribution of the normal stresses in the clamped cross-section from finite element results.

The abscissa contains the normal stress (tensile stresses are positive, compressive stresses are negative), while the ordinate axis represents the full thickness of the specimen ($y \in$ [-1.36 mm, +1.36 mm]). At cycle N = 1, the stress distribution is symmetric with respect to the midplane. Of course, due to the fact that more than one element is used through the thickness and the Bernoulli assumption is no longer imposed, the stress distribution at cycle N = 1 is no longer linear, although no damage is present at that time. Also, the presence of the clamping plates disturbs the stress state at the surface

near the fixation. When damage is initiating, the tensile stresses in the outermost layers are relaxed and load is transferred towards the inner layers. Because the damage law assumes that there is no damage growth at the compressive side, the peak tensile stresses are moving towards the compression side and the neutral fibre is moving down. It is very important to note that, for each given cycle, the plotted stress distribution in Figure 6 results from the equilibrium stress state of the whole composite specimen for the applied imposed displacement u_{max} and the stiffness distribution which is governed by the residual stiffness model (Equation (1)). The change of the stress state in each Gauss-point during fatigue life is governed exclusively by the stiffness degradation E/E_0 which can vary from point to point, because the stress amplitude $\Delta\sigma$ in the expression dD/dN might be different for each Gauss-point considered. Once the fatigue damage model has been established and the first cycle has been simulated, the initial stresses are known for each Gauss-point and the only driving force for further degradation is the residual stiffness model.
In that sense there is a major difference with the fatigue behaviour of metals. The only observable damage in metals is a crack, which grows in a self replicating manner, as can be described by fracture mechanics. As the stiffness of a metal remains quasi unaffected, the linear relation between stress and strain remains valid, and the fatigue process can be simulated in most common cases by one linear elastic analysis and linear fracture mechanics.

This brings about an important result: due to the growth of damage and the degradation of the bending stiffness, there is a continuous redistribution of the stresses in the cross-section, especially near the fixation where damage growth is dominant. The position of the 'neutral fibre' (according to its definition in the classical beam theory) does not remain in the middle of the cross-section, but tends to move towards the compression side and transfers the load to that zone. This has also been observed from the optical micrographs of the specimens.
The damage law (Equation (1)) has been applied to the fatigue behaviour of the $[\#0°]_8$ specimens as well, but the predictions were less accurate, because the fatigue damage mechanisms are different for both stacking sequences. Indeed, with the $[\#0°]_8$ specimens, only one direction of the glass fabric fibres are carrying load and due to the higher stiffness, the stresses are higher and fibre fracture is occurring. For the $[\#45°]_8$, the principal directions of the fabric are in an angle of 45° with the loading direction, and shear stresses are dominant leading to fibre/matrix debonding and matrix crazing. Especially when the displacement ratio R_d equals zero, the cracks are closed again and as explained by Wevers et al. [39,40], crack closure can lead to additional damage in the form of small matrix cracks at a ±45° angle to the full thickness matrix cracks and small size delaminations.

Currently, effort is made to develop new fatigue damage models which can cope with these different damage mechanisms.

7. Conclusions

This paper has discussed several topics concerning fatigue damage modelling of fibre-reinforced composite specimens. Accurate calculating of the laminar stress fields and modelling of the boundary conditions have been paid attention to.
The classes of fatigue models were discussed in general and the implementation of a residual stiffness model in particular. The finite element method, developed by the authors, simulates one loading cycle and then jumps over a certain number of cycles towards a new loading cycle where stiffness properties and damage distribution are altered, taking into account the effect on damage and stiffness of the cycles in between the two simulated loading cycles. The finite element implementation proves to be capable of simulating the observed force-cycle histories and stress redistribution due to stiffness degradation.

Acknowledgements

The author W. Van Paepegem gratefully acknowledges his finance through a grant of the Fund for Scientific Research – Flanders (F.W.O.), and the advice and technical support of the SAMTECH company. The authors also express their gratitude to Syncoglas for their support and technical collaboration.

References

[1] Mallick, P.K. (1997). Composites Engineering Handbook. New York, Marcel Dekker Inc.
[2] Herakovich, C.T. (1998). Mechanics of fibrous composites. New York, John Wiley & Sons, Inc.
[3] Reddy, J.N. (1997). Mechanics of Laminated Composite Plates. Theory and Analysis. Florida, CRC Press.
[4] Sol, H. and de Wilde, W.P. (1988). Identification of elastic properties of composite materials using resonant frequencies. In : Brebbia, C.A., de Wilde, W.P. and Blain, W.R. (eds.). Proceedings of the International Conference "Computer Aided Design in Composite Material Technology", Southampton, 1988, Springer-Verlag, pp. 273-280.
[5] Sol, H. (1990). Identification of the complex moduli of composite materials by a mixed numerical/experimental method. In : de Wilde, W.P. and Blain, W.R. (eds.). Proceedings of the second International Conference on Computer Aided Design in Composite Material Technology, Brussels, 25-27 April 1990, Springer-Verlag, pp. 267-279.
[6] Fujii, T., Amijima, S. and Okubo, K. (1993). Microscopic fatigue processes in a plain-weave glass-fibre composite. Composites Science and Technology, 49, 327-333.
[7] Schulte, K., Reese, E. and Chou, T.-W. (1987). Fatigue behaviour and damage development in woven fabric and hybrid fabric composites. In : Matthews, F.L., Buskell, N.C.R., Hodgkinson, J.M. and Morton,

J. (eds.). Sixth International Conference on Composite Materials (ICCM-VI) & Second European Conference on Composite Materials (ECCM-II) : Volume 4. Proceedings, 20-24 July 1987, London, UK, Elsevier, pp. 4.89-4.99.

[8] Hansen, U. (1997). Damage development in woven fabric composites during tension-tension fatigue. In : Andersen, S.I., Brøndsted, P., Lilholt, H., Lystrup, Aa., Rheinländer, J.T., Sørensen, B.F. and Toftegaard, H. (eds.). Polymeric Composites - Expanding the Limits. Proceedings of the 18th Risø International Symposium on Materials Science, 1-5 September 1997, Roskilde, Denmark, Risø International Laboratory, pp. 345-351.

[9] Ferry, L., Gabory, D., Sicot, N., Berard, J.Y., Perreux, D. and Varchon, D. (1997). Experimental study of glass-epoxy composite bars loaded in combined bending and torsion loads. Fatigue and characterisation of the damage growth. In : Degallaix, S., Bathias, C. and Fougères, R. (eds.). International Conference on fatigue of composites. Proceedings, 3-5 June 1997, Paris, France, La Société Française de Métallurgie et de Matériaux, pp. 266-273.

[10] Herrington, P.D. and Doucet, A.B. (1992). Progression of bending fatigue damage around a discontinuity in glass/epoxy composites. Journal of Composite Materials, 26(14), 2045-2059.

[11] Chen, A.S. and Matthews, F.L. (1993). Biaxial flexural fatigue of composite plates. In : Miravete, A. (ed.). ICCM/9 Composites : properties and applications. Volume VI. Proceedings of the Ninth International Conference on Composite Materials, 12-16 July 1993, Madrid, Spain, Woodhead Publishing Limited, pp. 899-906.

[12] Manger, C.I.C., Ogin, S.L., Smith, P.A. and Greaves, R.P. (1997). Damage development in plain weave GFRP. In : Scott, M.L. (ed.). ICCM/11 Volume V : Textile composites and characterisation. Proceedings of the Eleventh International Conference on Composite Materials (ICCM/11), 14-18 July 1997, Gold Coast, Queensland, Australia, Woodhead Publishing Limited, pp. V.58-V.66.

[13] Reddy, J.N. and Robbins, D.H. Jr. (1994). Theories and computational models for composite laminates. Applied Mechanics Review, 47(6), 147-167.

[14] Noor, A.K. and Burton, W.S. (1989). Assessment of shear deformation theories for multilayered composite plates. Applied Mechanics Review, 42(1), 1-13.

[15] Reddy, J.N. (1980). A penalty plate-bending element for the analysis of laminated anisotropic composite plates. International Journal for Numerical Methods in Engineering, 15, 1187-1206.

[16] Reddy, J.N. (1989). On refined computational models of composite laminates. International Journal for Numerical Methods in Engineering, 27, 361-382.

[17] Robbins, D.H. and Reddy, J.N. (1993). Modelling of thick composites using a layerwise laminate theory. International Journal for Numerical Methods in Engineering, 36, 655-677.

[18] Reddy, J.N. (1987). A generalization of two-dimensional theories of laminated composite plates. Communications in Applied Numerical Methods, 3, 173-180.

[19] Reddy, J.N. (1993). An evaluation of equivalent-single-layer and layerwise theories of composite laminates. Composite Structures, 25, 21-35.

[20] Naik, N.K., Chandra Sekher, Y. and Meduri, S. (2000). Damage in woven-fabric composites subjectes to low-velocity impact. Composites Science and Technology, 60, 731-744.

[21] Highsmith, A.L. and Reifsnider, K.L. (1982). Stiffness-reduction mechanisms in composite laminates. In : Reifsnider, K.L. (ed.). Damage in composite materials. ASTM STP 775. American Society for Testing and Materials, pp. 103-117.

[22] Sendeckyj, G.P. (1981). Fitting models to composite materials fatigue data. In : Chamis, C.C. (ed.). Test methods and design allowables for fibrous composites. ASTM STP 734. American Society for Testing and Materials, pp. 245-260.

[23] Hahn, H.T. and Kim, R.Y. (1975). Proof testing of composite materials. Journal of Composite Materials, 9, 297-311.

[24] Chou, P.C. and Croman, R. (1978). Residual Strength in Fatigue Based on the Strength-Life Equal Rank Assumption. Journal of Composite Materials, 12, 177-194.

[25] Harris, B. (1985). Fatigue behaviour of polymer-based composites and life prediction methods. AIB-Vinçotte Leerstoel, 2 maart 1995, Belgium, Nationaal Fonds voor Wetenschappelijk Onderzoek, 28 p.

[26] Hahn, H.T. and Kim, R.Y. (1976). Fatigue behaviour of composite laminates. Journal of Composite Materials, 10, 156-180.

[27] O'Brien, T.K. and Reifsnider, K.L. (1981). Fatigue damage evaluation through stiffness measurements in boron-epoxy laminates. Journal of Composite Materials, 15, 55-70.

[28] Hwang, W. and Han, K.S. (1986). Cumulative damage models and multi-stress fatigue life prediction. Journal of Composite Materials, 20, 125-153.

[29] Schaff, J.R. and Davidson, B.D. (1997). Life prediction methodology for composite structures. Part I - Constant amplitude and two-stress level fatigue. Journal of Composite Materials, 31(2), 128-157.

[30] Hashin, Z. (1985). Cumulative damage theory for composite materials: residual life and residual strength methods. Composites Science and Technology, 23, 1-19.

[31] Whitworth, H.A. (2000). Evaluation of the residual strength degradation in composite laminates under fatigue loading. Composite Structures, 48(4), 261-264.

[32] Yang, J.N., Jones, D.L., Yang, S.H. and Meskini, A. (1990). A stiffness degradation model for graphite/epoxy laminates. Journal of Composite Materials, 24, 753-769.

[33] Yang, J.N., Lee, L.J. and Sheu, D.Y. (1992). Modulus reduction and fatigue damage of matrix dominated composite laminates. Composite Structures, 21, 91-100.

[34] Kedward, K.T. and Beaumont, P.W.R. (1992). The treatment of fatigue and damage accumulation in composite design. International Journal of Fatigue, 14(5), 283-294.

[35] Van Paepegem, W. and Degrieck, J. (2000). Experimental setup for the bending fatigue of fibre-reinforced composite materials. In: Bache, M.R. et al. (eds.). Fatigue 2000 : Fatigue & Durability assessment of materials, components and structures. Proceedings. Cambridge, 10-12 April 2000, Chameleon Press Ltd., pp. 115-122.

[36] Van Paepegem, W. and Degrieck, J. (2001). Experimental setup for and numerical modelling of bending fatigue experiments on plain woven glass/epoxy composites. Composite Structures, 51(1), 1-8.

[37] Sidoroff, F. and Subagio, B. (1987). Fatigue damage modelling of composite materials from bending tests.

In : Matthews, F.L., Buskell, N.C.R., Hodgkinson, J.M. and Morton, J. (eds.). Sixth International Conference on Composite Materials (ICCM-VI) & Second European Conference on Composite Materials (ECCM-II) : Volume 4. Proceedings, 20-24 July 1987, London, UK, Elsevier, pp. 4.32-4.39.

[38] Van Paepegem, W. and Degrieck, J. (2000). Numerical modelling of fatigue degradation of fibre-reinforced composite materials. In: Topping, B.H.V. (ed.). Proceedings of the Fifth International Conference on Computational Structures Technology. Volume F: Computational Techniques for Materials, Composites and Composite Structures, Leuven, 6-8 September 2000, Civil-Comp Press, pp. 319-326.

[39] Wevers, M., Verpoest, I., Aernoudt, E. and De Meester, P. (1987). Fatigue damage development in carbon fiber reinforced epoxy composites: correlation between the stiffness degradation and the growth of different damage types. In : Matthews, F.L. et al. (eds.). Sixth International Conference on Composite Materials (ICCM-VI) & Second European Conference on Composite Materials (ECCM-II) : Volume 4. Proceedings, 20-24 July 1987, London, UK, Elsevier, pp. 4.114-4.128.

[40] Wevers, M., Verpoest, I. and De Meester, P. (1990). Is crack closure due to fatigue loading causing more damage in carbon fibre reinforced epoxy composites ? In : Füller, J. et al. (eds.). ECCM-4 : Developments in the science and technology of composite materials. Proceedings of the Fourth European Conference on Composite Materials, 25-28 September, Stuttgart, Germany, Elsevier, pp. 181-188.